Computer Physics Communications 201 (2016) 77–84

Contents lists available at ScienceDirect

Computer Physics Communications

journal homepage: www.elsevier.com/locate/cpc

A Markov Chain-based quantitative study of angular distribution of photons through turbid slabs via isotropic light scattering

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ARTICLE INFO

Article history:
Received 19 July 2015
Received in revised form
17 December 2015
Accepted 26 December 2015
Available online 6 January 2016

Keywords:
Multiple scattering
Markov processes
Monte Carlo methods

ABSTRACT

This paper describes a quantitative approach to approximate multiple scattering through an isotropic turbid slab based on Markov Chain theorem. There is an increasing need to utilize multiple scattering for optical diagnostic purposes; however, existing methods are either inaccurate or computationally expensive. Here, we develop a novel Markov Chain approximation approach to solve multiple scattering angular distribution (AD) that can accurately calculate AD while significantly reducing computational cost compared to Monte Carlo simulation. We expect this work to stimulate ongoing multiple scattering research and deterministic reconstruction algorithm development with AD measurements.

1. Introduction

Particle and spray diagnostics has attracted increasing interest in a variety of fields, such as spray analysis for combustion [1,2], particulate matter (PM) monitoring and soot emission control [3], medical diagnostics [4,5], and remote sensing [6]. Among the diagnostic methods, optical diagnostics based on scattering are emerging due to their potential for instantaneous measurements. A significant amount of research in the area falls into the category of Mie scattering where the size of scattering particles is close to the wavelength of the incidental light. Important parameters, such as optical depth (OD) of the spray or turbid slab, spray particle size, and spray structure, can be inferred via the output light signal that goes through Mie Scattering, for example, light intensity attenuation or diffusion. Common practices for understanding light scattering process include solving a set of wave equations or a set of simplified relations including radiative transfer equations (RTEs) [7]. However, significant challenges arise when solving these equations, especially under complex realistic conditions; therefore, RTE application is usually limited to 1D. Numerous efforts to approximate the light scattering process have been attempted and good approximations can be found for single scattering regime (OD < 1) and multiple scattering regime (OD $>$ 10). However, the light scattering approximation with an OD between 1 and 10 (intermediate regime) remains a challenge [8]. Common methods for simulating light scattering within the intermediate regime include random walk (RW) approximation, which provides simplified mathematical formation to predict transmitted photon properties, and Monte Carlo simulation, which relies on repeated random sampling (ray tracing) to obtain a statistically meaningful result.

The angular distribution (AD) of multiple scattering photons has been utilized in many areas such as lens selection [9] and atmosphere remote sensing [10]. Recently, AD shows great research potential in optical diagnostics, for example, to determine particle sizes in a turbid slab [11]. In order to reconstruct the particle size distribution or optical depth distribution, especially in a complex turbid slab, a robust inversion algorithm is highly desirable. Researchers have used many inversion algorithms for optical diagnostic reconstructions, such as simulated annealing (SA) [12,13] and computed tomography (CT) algorithms [14]. Most reconstruction algorithms require a fast evaluation of the difference between simulations and measurements in the form of a cost function. However, a fast and reliable calculation of AD, especially in the intermediate regime is not readily available. Existing literature have investigated AD using RTEs [15] but its use is limited by complexity. Gandjbakhche et al. [16] utilized lattice random walk theory to predict several observations including transmitted photon location distribution, transmitted path length distribution, and total transmission. However, the RW approximation has limitations such as requiring that the scattering medium be uniform, only being applicable at specific ODs, and lack of an available explicit expression for AD. Monte Carlo simulation [8] is capable of finding AD but the computational cost is prohibitively expensive for reconstruction applications, thus rendering it less desirable. A previous work [17] attempted to use an analytical, closed-form method to calculate

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http://dx.doi.org/10.1016/j.cpc.2015.12.020

AD for isotropic scattering. Reasonable agreements were found between the analytical solutions and Monte Carlo simulations under different conditions. However, the application of the work is limited because $Q(z)$, which was defined as the fraction of the transmitted photons that undergo their last scattering event at z, was an empirical prediction and no rigorous derivation was given.

These considerations inspire our Markov Chain approximation approach which can both (1) significantly reduce the computation cost for each AD evaluation and (2) obtain high fidelity AD simulations for inversion algorithms. The Markov Chain approximation can also be used for more complex optical diagnostics applications, such as anisotropic scattering investigations or determining spatial distribution of transmitted photons. However, in this paper, we will limit the scope of our discussion to isotropic AD analysis. Expanded investigations will be covered by future work. This paper is organized as follows: Section 2 briefly introduces the fundamentals of Mie Scattering and Markov Chain approximation and describes the application of the Markov Chain method to Mie Scattering. An introduction of Monte Carlo simulation for light scattering is also included in this section. Section 3 demonstrates the results obtained by the Markov Chain method and compares the results with those obtained by Monte Carlo simulation. Section 4 incorporates an error analysis and discusses future Markov Chain approximation applications. Finally, Section 5 summarizes this paper.

2. Scattering and approximation approaches

2.1. Photon transport fundamentals

Fig. 1 demonstrates an example of light scattering through a turbid slab. We assume an infinitely large turbid slab in the x and y directions and fixed thickness. Light (photons) propagates into the turbid slab (scattering medium) in the $-z$ direction. The photons then may be scattered and propagate in another direction then be scattered repeatedly until they leave the turbid slab. If the photons propagate through the turbid slab without any scattering (scenario (1)), we call them ballistic photons. These photons propagate the shortest distance possible through the slab. If the photons scatter and leave the turbid slab from the bottom plane, as can be seen in scenario (2), we define them as transmitted photons. Ballistic photons are usually considered to be transmitted photons, but within the scope of this paper, we exclude them because ballistic photon characteristics are more predictable and complex analysis is not required. The remaining photons that return to the top plane after a series of scattering, as can be seen in scenario (3), are defined as reflected photons. Because the slab is infinitely large, photons will be either transmitted or reflected, and eventually leave the turbid slab if we assume there is no absorption effect. The angle θ formed by z-axis and the propagation direction of photons are defined as propagation angles. Specifically, if the photons are to exit the slab from the exit plane, we call the angle θ the transmitted angle; and if photons exit from the entrance plane, we call the angle θ the reflected angle.

When photons propagate in scattering mediums, there are two factors that determine their scattering characteristics. The first factor is how the propagation direction of the photons changes when scattering takes place. For Mie scattering, the change of propagation direction is determined by the phase function, which is a complex function of several parameters such as the diameter of the particles, wavelength of the incident light, refractive index of the particles, etc. In this study we focus on isotropic scattering, i.e., the photon has an equal chance to propagate in every direction and no practical phase function was applied. The second factor is how far the photon propagates until the next scattering event. The propagation distance is determined by the optical depth (OD)

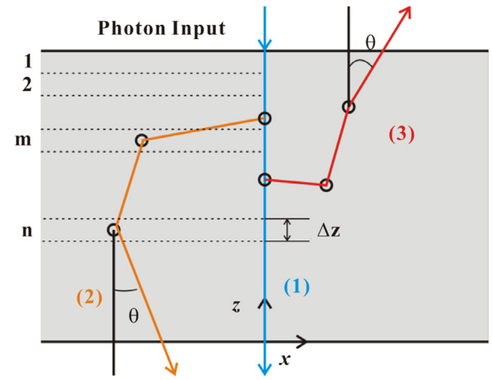

Fig. 1. Schematic of the light scattering problem.

traversed by the photon. The probability of the photon propagating through a free path length of l_{fp} can be expressed by [18]:

$$l_{fp} = -\frac{\ln \xi}{\mu_e} \tag{1}$$

where ξ is a random number that has a uniform probability between 0 and 1, and μ_e is the extinction coefficient. Also note that:

$$OD_{fp} = l_{fp} \cdot \mu_e. \tag{2}$$

We define OD_{fp} as the optical depth that the photon has propagated through between two adjacent scattering events. Then we can easily derive the following equation:

$$P(OD_{fp}) = \exp(-OD_{fp}) \tag{3}$$

where $P(OD_{fp})$ is the probability that a photon propagates OD_{fp} between two adjacent scattering events. If we discretize the turbid slab into Z layers with equal thickness Δz, as shown in Fig. 1 (note that the OD in each layer might be different), the probability that a photon scattered in layer m takes the next scattering event in layer n at a transmitted angle of θ can then be expressed by:

$$\begin{aligned} P(m, n, \theta) &= \Phi(\theta) \cdot \exp(-OD_{fp}) \\ &= \frac{\sin\theta}{2} \cdot \exp\left(-\sum_{i=m}^{n} \frac{\Sigma_{s,i} \cdot \Delta z}{\cos\theta}\right) \end{aligned} \tag{4}$$

where $\Sigma_{s,i}$ is the scattering coefficient of layer i for a photon propagating at $\theta = 0$ (i.e. in $-z$ direction) and $OD_{fp} = \sum_{i=m}^{n}(\Sigma_{s,i} \cdot \Delta z)$. $\Phi(\theta) = \sin\theta/2$ is the phase function for isotropic scattering. Finally, the probability for a photon to propagate from layer m to layer n at any propagation angle θ is given by:

$$P(m, n) = \int_0^{\pi/2} \frac{\sin\theta}{2} \cdot \exp\left(-\sum_{i=m}^{n} \frac{\Sigma_{s,i} \cdot \Delta z}{\cos\theta}\right) d\theta. \tag{5}$$

Eq. (5) is a crucial relationship because it specifies the transition probability from layer m (or state m) to layer n (or state n). Furthermore, photon scattering has a "memoryless" property, which means the next scattering event can only be determined from the current scattering event. Thus, *a priori* knowledge of the scattering events before the current scattering event is not required. Markov Chain theory can now be used as a robust tool to approximate scatterings by combining defined transition probability with the memoryless feature.

2.2. Absorbing Markov Chain theorem

The Markov Chain theorem has found wide applications in decision making, queueing theory, physics, games, internet applications, etc. [19]. A transition matrix, P is used to describe the