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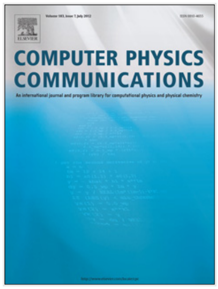

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On the efficient numerical solution of lattice systems with low-order couplings

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Abstract

We apply the Quasi Monte Carlo (QMC) and recursive numerical integration methods to evaluate the Euclidean, discretized time path-integral for the quantum mechanical anharmonic oscillator and a topological quantum mechanical rotor model. For the anharmonic oscillator both methods outperform standard Markov Chain Monte Carlo methods and show a significantly improved error scaling. For the quantum mechanical rotor we could, however, not find a successful way employing QMC. On the other hand, the recursive numerical integration method works extremely well for this model and shows an at least exponentially fast error scaling.

Keywords: recursive numerical integration, quasi monte carlo, quantum mechanical rotor, anharmonic oscillator, lattice systems, low order couplings

1. Introduction

Markov Chain Monte Carlo (MCMC) is the method of choice for simulations of quantum field theories or systems in statistical physics. The advantage of MCMC is that it can be applied very generally to many physical models. It allows to compute expectation values of physical observables $\langle O\rangle$ with an error Δ which scales only as $\Delta \propto 1/\sqrt{N}$, however, where N is the number of samples. This error scaling law leads to a very large numerical effort if another significant digit in the accuracy of an observable is needed.

In quantum field theory, in particular quantum chromodynamics (QCD) - our theory of the strong interaction between quarks and gluons - very significant progress has been achieved in the last years through improvements of the MCMC methods used; see, e.g., ref. [1] for an overview. But, even though lattice QCD simulations of the theory could be accelerated substantially, computations typically run several months or even years on state of the art supercomputers. In

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