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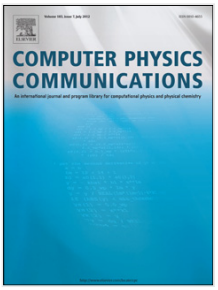

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Bratu's Problem: a Novel Approach using Fixed-Point Iterations and Green's Functions

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Abstract

In this article, the one-dimensional non-linear Bratu's boundary value problem is solved via a novel approach that combines Green's function and fixed point iterative schemes, such as Picard's and Krasnoselskii-Mann's. The convergence of the introduced iterative algorithm is proved using the contraction principle. The method is supported by considering a number of numerical examples that correspond to different cases of eigenvalues. The procedure underlying the strategy reduces calculations and provides highly accurate results in comparison with the exact solution and/or numerical solutions provided in the literature. The current method overcomes the difficulty of treating the problem for eigenvalues near and at the critical value, such as $\lambda = 3$ and $\lambda = 3.51$, and handles them reliably and very efficiently.

1 Introduction

In this paper, we introduce an iterative algorithm to solve the one-dimensional Bratu's nonlinear boundary value problem given by

$$-u''(t) = \lambda\, e^{u(t)}, \tag{1.1}$$

subject to the boundary conditions

$$u(0) = 0, \quad \& \quad u(1) = 0, \tag{1.2}$$

where $\lambda > 0$.

The exact solution of (1.1)-(1.2) is given by [5]:

$$u(t) = -2\ln\left[\frac{\cosh\left(0.5(t-0.5)\theta\right)}{\cosh(\theta/4)}\right], \tag{1.3}$$

provided that θ is the solution of $\theta = \sqrt{2\lambda}\cosh(\theta/4)$. The parameter λ characterizes the number of possible solutions for the Bratu problem which has a critical turning point λ_c numerically approximated by 3.513830719. For this point, the problem has a unique solution whereas there exists two solutions for $\lambda < \lambda_c$ and no solution for $\lambda > \lambda_c$.

The proposed approach is based on embedding an integral operator, which

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