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Implicit predictor–corrector central finite difference scheme for the equations of magnetohydrodynamic simulations

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ABSTRACT

Nowadays most of supercomputers are based on the frame of PC cluster; therefore, the efficiency of parallel computing is of importance especially with the increasing computing scale. This paper proposes a high-order implicit predictor–corrector central finite difference (iPCCFD) scheme and demonstrates its high efficiency in parallel computing. Of special interests are the large scale numerical studies such as the magnetohydrodynamic (MHD) simulations in the planetary magnetosphere. An iPCCFD scheme is developed based on fifth-order central finite difference method and fourth-order implicit predictor–corrector method in combination with elimination-of-the-round-off-errors (ERE) technique. We examine several numerical studies such as one-dimensional Brio–Wu shock tube problem, two-dimensional Orszag–Tang vortex system, vortex type K–H instability, kink type K–H instability, field loop advection, and blast wave. All the simulation results are consistent with many literatures. iPCCFD can minimize the numerical instabilities and noises along with the additional diffusion terms. All of our studies present relatively small numerical errors without employing any divergence-free reconstruction. In particular, we obtain fairly stable results in the two-dimensional Brio–Wu shock tube problem which well conserves $\nabla \cdot \mathbf{B} = 0$ throughout the simulation. The ERE technique removes the accumulation of roundoff errors in the uniform or non-disturbed system. We have also shown that iPCCFD is characterized by the high order of accuracy and the low numerical dissipation in the circularly polarized Alfvén wave tests. The proposed iPCCFD scheme is a parallel-efficient and high precision numerical scheme for solving the MHD equations in hyperbolic conservation systems.

1. Introduction

Magnetohydrodynamic (MHD) modeling plays an important role on space physics and astrophysics simulation studies. For example, numerical solar wind models [1–3] based on the MHD theory are currently the only self-consistent mathematical descriptions that are capable of bridging the physical system from many Astronomical Units (A.U.) near the Sun to well beyond the Earth's orbit. Although MHD theory can only approach a partial approximation to actual plasma behavior, it has successfully simulated many important plasma properties in space [4–6]. Numerous space physics and astrophysics MHD numerical problems focus on solving hyperbolic time-dependent partial differential equations. In the past decades majority of numerical schemes developed with significant considerations of the computing speed, memory capacity, and accuracy due to the limitations of the computing capacity and speed: for instance, Lax–Friedrichs scheme [7], Lax–Wendroff scheme [8], and the follow-on upwind differencing scheme [9–11]. With the evolution of high performance computing the recent developments of simulation models concentrate on high-order numerical schemes.

However, numerical instabilities of the high-order schemes still limit the progress of these models, which cannot be significantly improved until Weighted Essentially Non-Oscillatory (WENO) scheme developed by Liu, Osher, and Chan [12]. The WENO scheme uses fewer grid points to obtain the non-oscillatory, high resolution, and stable simulation results, such as in the large amplitude discontinuities and shocks. Other high-order schemes

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have since evolved [13–16] from the WENO schemes [12]. The high-order WENO numerical schemes for the MHD simulations can be separated into two catalogs: the WENO scheme with the MHD Riemann solvers [17] and the Central WENO (CWENO) scheme without the MHD Riemann solvers [18,19]. The WENO scheme requires the evaluation of the MHD eigensystem at each integer grid point in order to deal with the flux splitting. In comparison, the CWENO scheme does not demand the computation of eigenvalues and therefore saves the computing time: nevertheless, the calculations of the derivative along the x direction need the input from the information in the y direction in the 2D case [18,19]. Moreover, an additional divergence-free reconstruction of the magnetic field $\mathbf{B}$ needs to be carried out to retain the divergence of $\mathbf{B}$ conserved in several MHD numerical simulation schemes [17,19].

Although implicit numerical methods are unconditionally stable, it significantly increases the complexity of the program and the computing time, meanwhile, it is difficult to enhance the parallel computing efficiency because of inverting large matrices. Since nowadays most of supercomputers are based on the frame of PC cluster, the consideration of the efficiency in parallel computing would become more important especially with the increasing computing scale. For this tendency, an implicit numerical method with characteristics of high-efficient parallel computing, high numerical precision, low dispersion, and low dissipation is strongly desired. In this paper we employ the implicit predictor–corrector method for time stepping. Although the iterations of the predictor–corrector method result in a large amount of computation, the increase of the computing efficiency can be expected because no matrices are required to be solved. In addition, we use the high-order central finite difference method [20] to solve the spatial derivative at each integer grid point to incorporate with the implicit time stepping method. All of the computations in the non-staggered finite difference method are on the integer grid points such that the structure is simple, easy programming, and simple to be replaced by a numerical method with different orders depending on our demand of the resolutions. In this paper we use fifth-order central finite difference method and fourth-order implicit predictor–corrector method. We denote this method as the high-order implicit predictor–corrector central finite difference (iPCCFD) scheme. The Elimination-of-the-Roundoff-Error (ERE) technique [21] is also implemented to minimize the numerical errors in our approach. This scheme enhances the numerical precision and minimizes the numerical error (particularly in the non-disturbed system), thereby approximately conserving $\nabla \cdot \mathbf{B} = 0$ without using any divergence-free reconstruction. Although the high-order finite difference method, implicit time stepping method, and ERE technique of iPCCFD scheme highly increase the amount of computation, the efficiency of parallel computing is strongly strengthened because no band matrices are required to be solved. Such an advantage allows us to use the larger supercomputer to simulate the more precise and the larger scale simulations. For instance, we can use the high-resolution 3D global MHD code to simulate the plasma behaviors in the large scale system, which requires a large amount of computations such as in the interplanetary magnetosphere.

This paper is organized as follows. In Section 2 we introduce the governing equations of the MHD simulations. The numerical scheme and ERE technique are detailed in Section 3. In Section 4 we demonstrate the performance of the high-order iPCCFD scheme by showing the numerical results of 1D and 2D Brio–Wu shock tube problems. The simulations of the 2D Orszag–Tang vortex system, vortex-type and kink-type Kelvin–Helmholtz instabilities, field loop advection, and blast wave test are also presented in Section 4. In addition, we quantitatively examine the accuracy of iPCCFD scheme in the problems of 1D and 3D circularly polarized Alfvén wave propagation. The numerical dissipation in the 2D run is also examined. Finally, we discuss and summarize the advantage of iPCCFD scheme in Section 5.

2. The governing equations

In this paper the MHD equations with diffusions are

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}), \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{V}) + \nabla \cdot \left[\rho \mathbf{V}\mathbf{V} + \left(p + \frac{B^2}{2}\right)\mathbf{1} - \mathbf{B}\mathbf{B}\right] = \eta_{V_i} \rho \nabla^2 \mathbf{V}, \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho V^2 + \frac{p}{\gamma - 1} + \frac{B^2}{2}\right) + \nabla \cdot \left[\left(\frac{1}{2}\rho V^2 + \frac{\gamma p}{\gamma - 1} + B^2\right)\mathbf{V} - \mathbf{B}\mathbf{B} \cdot \mathbf{V}\right] = \eta_{T_i} \rho \nabla^2 \left(\frac{p}{\rho}\right), \quad \text{and} \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{5}$$

Here γ is the ratio of specific heats. $\mathbf{B}$, $\mathbf{V}$, ρ, and p are the magnetic field, the plasma bulk velocity, the plasma mass density, and the plasma thermal pressure, respectively. The diffusion terms are added in Eqs. (3) and (4) in order to stabilize the numerical algorithm, where η_{V_i} and η_{T_i} are the diffusion coefficients. The Eqs. (1)–(4) can be written as the following equation set in Cartesian coordinates

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \frac{\partial \mathbf{H}}{\partial z} + \boldsymbol{\eta}\frac{\partial^2 \mathbf{D}_x}{\partial x^2} + \boldsymbol{\eta}\frac{\partial^2 \mathbf{D}_y}{\partial y^2} + \boldsymbol{\eta}\frac{\partial^2 \mathbf{D}_z}{\partial z^2} = 0, \tag{6}$$

where

$$\mathbf{U} \equiv \begin{bmatrix} B_x \\ B_y \\ B_z \\ \rho \\ M_x \\ M_y \\ M_z \\ E \end{bmatrix},$$

$$\mathbf{F} \equiv \begin{bmatrix} 0 \\ B_y V_x - B_x V_y \\ B_z V_x - B_x V_z \\ \rho V_x \\ \rho {V_x}^2 + \left(p + \frac{B^2}{2}\right) - {B_x}^2 \\ \rho V_x V_y - B_x B_y \\ \rho V_x V_z - B_x B_z \\ \left(\frac{1}{2}\rho V^2 + \frac{\gamma p}{\gamma - 1} + B^2\right) V_x - B_x (B_x V_x + B_y V_y + B_z V_z) \end{bmatrix},$$

$$\mathbf{G} \equiv \begin{bmatrix} B_x V_y - B_y V_x \\ 0 \\ B_z V_y - B_y V_z \\ \rho V_y \\ \rho V_y V_x - B_y B_x \\ \rho {V_y}^2 + \left(p + \frac{B^2}{2}\right) - {B_y}^2 \\ \rho V_y V_z - B_y B_z \\ \left(\frac{1}{2}\rho V^2 + \frac{\gamma p}{\gamma - 1} + B^2\right) V_y - B_y (B_x V_x + B_y V_y + B_z V_z) \end{bmatrix},$$

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