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Hyperbolic graph generator☆

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ABSTRACT

Networks representing many complex systems in nature and society share some common structural properties like heterogeneous degree distributions and strong clustering. Recent research on network geometry has shown that those real networks can be adequately modeled as random geometric graphs in hyperbolic spaces. In this paper, we present a computer program to generate such graphs. Besides real-world-like networks, the program can generate random graphs from other well-known graph ensembles, such as the soft configuration model, random geometric graphs on a circle, or Erdős–Rényi random graphs. The simulations show a good match between the expected values of different network structural properties and the corresponding empirical values measured in generated graphs, confirming the accurate behavior of the program.

Program summary

Program title: Hyperbolic graph generator

Catalogue identifier: AEXC_v1_0

Program summary URL: http://cpc.cs.qub.ac.uk/summaries/AEXC_v1_0.html

Program obtainable from: CPC Program Library, Queen's University, Belfast, N. Ireland

Licensing provisions: GNU General Public License, version 3

No. of lines in distributed program, including test data, etc.: 101190

No. of bytes in distributed program, including test data, etc.: 771660

Distribution format: tar.gz

Programming language: C++.

Computer: Any.

Operating system: Any.

Classification: 6.3, 4.13, 23.

Nature of problem: Generation of graphs in hyperbolic spaces.

Solution method: Implementation based on analytical equations.

Additional comments: Can be used as a command-line tool or installed as a library to support more complex software.

Running time: Depends on the number of nodes. A few seconds for the graph in the example provided.

1. Introduction

The interactions between components of a complex system are often represented as a network. This modeling allows for rigorous mathematical treatment, and broadens our understanding of the system [1]. Many real networks possess common structural patterns, including, in the first place, heterogeneous (often

☆ This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

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Table 1
Regimes in the model.

γ \ T	0	$(0, \infty)$	∞
$[2, \infty)$	Hyperbolic RGGs	Soft hyperbolic RGGs	Soft configuration model
∞	Spherical RGGs	Soft spherical RGGs	Erdős–Rényi

power-law) distributions of node degrees [2], and strong clustering, i.e., higher numbers of triangular subgraphs than predicted by classical random graph models [3]. Recently introduced geometric graph models, based on the assumption that nodes in real networks are embedded in latent hyperbolic spaces [4,5], reproduce these common structural properties of real networks. Furthermore, these hyperbolic graphs replicate dynamical processes on top of real networks [6] and accurately predict missing links in them [7].

In this work we present a program to generate random hyperbolic graphs. This software implements and extends the network model introduced in [4]. Nodes are randomly sprinkled on a hyperbolic disk, and the probability of the existence of an edge (the connection probability) between two nodes is a function of the distance between the nodes in the hyperbolic space. Thus generated graphs have strong clustering, and node degree distributions in them are power laws. Moreover, other popular and well-studied random graph ensembles, namely the soft configuration model (SCM) [8], (soft) random geometric graphs (RGGs) on a circle [9,10], and Erdős–Rényi (ER) random graphs [11], appear as degenerate regimes in the model. Table 1 shows all the model regimes, the total of six. Each regime is defined by the values of only two parameters: γ, which is the expected exponent of the power-law degree distribution, and temperature T, the parameter controlling the strength of clustering in the network.

Researchers in different disciplines may benefit from the use of random hyperbolic graphs in their work. Yet the full implementation of the model and all its regimes is a tricky business, which involves dealing with some delicate details, due to a variety of internal parameters and their interactions over the six regimes. In Section 2–4 we describe the implementation details of the model, including how all the parameters are calculated in each regime. A good match between the values of expected graph properties and their observed values in generated graphs is confirmed in Section 5.

2. Input parameters and coordinates

2.1. Input parameters

The program input parameters are the number of nodes N, the target expected average degree $\bar{k}$ of the network, the target expected power-law exponent γ of the degree distribution, and temperature T. The combination of γ and T values will define the graph ensemble from which generated networks are sampled (Table 1).

Given the input parameters, the graph generation process consists of three steps:

1. Compute the internal parameters, such as the radius R of the hyperbolic disk occupied by nodes, as functions of the input parameter values, Sections 3 and 4.
2. Assign to all nodes their angular and radial coordinates on the hyperbolic plane, Section 2.2.
3. Connect each node pair by an edge with probability (the connection probability), which is a function of the coordinates of the two nodes, Sections 3 and 4.

2.2. Coordinate sampling

The assignment of node coordinates is done as follows in all the six regimes.

Angular coordinates θ of nodes are assigned by sampling them uniformly at random from interval $[0, 2\pi)$, i.e., the angular node density is uniform $\rho(\theta) = 1/(2\pi)$.

Radial coordinates $r \in [0, R]$, where R is the radius of the hyperbolic disk, are sampled from the following distribution, which is nearly exponential with exponent $\alpha > 0$,

$$\rho(r) = \alpha \frac{\sinh \alpha r}{\cosh \alpha R - 1} \approx \alpha e^{\alpha(r-R)}. \quad (1)$$

The calculation of internal parameter R is described in detail below; it is different in different regimes. Internal parameter α depends on the expected exponent γ of the power distribution $P(k) \sim k^{-\gamma}$ of nodes degrees k in generated graphs, and on the curvature of the hyperbolic space $\zeta = \sqrt{-K}$, which is set to $\zeta = 1$ by default. For temperatures $T \leq 1$, this relationship is given by

$$\gamma = 2\frac{\alpha}{\zeta} + 1, \quad (2)$$

while for $T > 1$ it becomes

$$\gamma = 2\frac{\alpha}{\zeta}T + 1. \quad (3)$$

To sample radial coordinates r according to the distribution in Eq. (1), the inverse transform sampling is used: first a random value U_i is sampled from the uniform distribution on [0, 1], and then the radial coordinate of node i is set to

$$r_i = \frac{1}{\alpha} \operatorname{acosh}\left(1 + (\cosh \alpha R - 1)\, U_i\right), \quad \text{for } i = 1, \ldots, N. \quad (4)$$

3. Regimes with finite $\gamma \geq 2$

3.1. $T \in (0, \infty)$: soft hyperbolic random geometric graphs

This is the most general regime in the model, from which all other regimes can be obtained as limit cases. The connection probability in this case is

$$p(x) = \frac{1}{1 + e^{\beta(\zeta/2)(x-R)}}, \quad (5)$$

where $\beta = 1/T$, and R is the radius of the hyperbolic disk occupied by nodes. The hyperbolic distance x between two nodes at polar coordinates (r, θ) and (r', θ') is given by

$$x = \frac{1}{\zeta} \operatorname{arccosh}\left(\cosh \zeta r \cosh \zeta r' - \sinh \zeta r \sinh \zeta r' \cos \Delta\theta\right), \quad (6)$$

where $\Delta\theta = \pi - |\pi - |\theta - \theta'||$ is the angular distance between the nodes. To calculate the expected degree of a node at radial coordinate r, without loss of generality its angular coordinate can be set to zero, $\theta = 0$, so that its expected degree can be written as

$$\bar{k}(r) = \frac{N}{\pi} \int_0^R \rho(r') \int_0^\pi p(x)\, d\theta' dr'. \quad (7)$$