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Numerical evaluation of multi-gluon amplitudes for High Energy Factorization ☆

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ABSTRACT

We present a program to evaluate tree-level multi-gluon amplitudes with up to two of them off-shell. Furthermore, it evaluates squared amplitudes summed over colors and helicities for up to six external gluons. It employs both analytic expressions, obtained via BCFW recursion, and numerical BCFW recursion. It has been validated numerically with the help of an independent program employing numerical Dyson–Schwinger recursion.

Program summary

Program title: AMP4HEF

Catalogue identifier: AEXF_v1_0

Program summary URL: http://cpc.cs.qub.ac.uk/summaries/AEXF_v1_0.html

Program obtainable from: CPC Program Library, Queen's University, Belfast, N. Ireland

Licensing provisions: Standard CPC licence, http://cpc.cs.qub.ac.uk/licence/licence.html

No. of lines in distributed program, including test data, etc.: 26563

No. of bytes in distributed program, including test data, etc.: 467441

Distribution format: tar.gz

Programming language: Fortran 2003, implemented at least as far as in gfortran- 4.6.

Computer: Any computer with the required Fortran compiler.

Operating system: Any operating system with the required Fortran compiler.

RAM: Negligible

Classification: 11.1.

Nature of problem: The numerical evaluation of matrix elements applicable in factorization prescriptions for the cross section calculation of hadron scattering processes that require off-shell partonic initial states.

Solution method: The generalization of the recursive method of Britto, Cachazo, Feng and Witten [1] to amplitudes with off-shell gluons [2] is applied, both to obtain expressions which are hard-coded in the program, and numerically.

Restrictions: Matrix element summed over color and spin can be evaluated for up to 6 external gluons with up to 2 of them off-shell. Color-ordered amplitudes can be evaluated essentially for an arbitrary number of external gluons, with up to 2 of them off-shell.

Running time: The evaluation of the matrix element for a single phase space point summed over colors and helicities takes 0.005ms for 4 gluons with 2 of them off-shell, 0.1ms for 5 gluons with 2 of them off-shell, and 2.7ms for 6 gluons with 2 of them off-shell. This is on a single 2.30 GHz Intel Core i7, without any compiler optimization.

☆ This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

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References:

[1] R. Britto, F. Cachazo, B. Feng, and E. Witten, Direct proof of tree-level recursion relation in Yang–Mills theory, Phys. Rev. Lett. 94 (2005) 181602, [hep-th/0501052].

[2] A. van Hameren, BCFW recursion for off-shell gluons, JHEP 1407 (2014) 138, [1404.7818].

1. Introduction

Factorization prescriptions are powerful tools to tame the complex calculations involving quantum chromodynamics (QCD) for scattering processes at collider experiments like at the Large Hadron Collider. They factorize contributions to cross sections according to the scales involved, and/or according to universality and accessibility via perturbation theory. Many factorization prescriptions are heuristic in nature, and some are proven, which means that perturbation theory and the treatment of possible singularities, along with the occurrence of large logarithms of ratios of scales, can be dealt with in a systematic manner.

Partonic scattering amplitudes form an essential ingredient in factorized calculations of cross sections for hadron collisions. Even though the partonic states are not physical, on-shell partonic amplitudes are well defined gauge invariant objects of the gauge theory QCD, using its Lagrangian and the Lehmann–Symanzik–Zimmermann reduction formula. Factorization embeds the non-physical scattering amplitudes into physical cross sections. Recently, it has been shown that scattering amplitudes involving any number of *off-shell* external gluons can also be defined in a rigorous manner [1]. Such amplitudes are relevant in factorization prescriptions requiring off-shell initial-state partons, like High Energy Factorization (HEF) [2,3]. Recent developments and calculations involving such factorization prescriptions can be found in [4–12].

Calculations employing collinear factorization, for which the scattering amplitudes are completely on-shell, have been automated to the end for arbitrary processes, with essentially arbitrary multiplicities, and within essentially arbitrary models of quantum field theory [13–22]. By now, developments are heading at reaching this status to next-to-leading order in perturbation theory. This includes one-loop amplitudes and real-radiation contributions with all the complications arising due to mass singularities and the highly non-trivial phase space integration [23–29].

HEF requires partonic scattering amplitudes with off-shell initial state partons, and automation of the calculation of these has not been established. Systematic formulations of their calculation have been established [30–34]. In this paper, we present a program to numerically evaluate tree-level multi-gluon scattering amplitudes with up to two off-shell gluons as function of the gluon momenta, squared and summed over colors of all gluons, and summed over the spins of the on-shell gluons. It evaluates them via color-ordered helicity amplitudes that are calculated using the generalization of Britto–Cachazo–Feng–Witten (BCFW) recursion [35,36], described in [37], to include off-shell gluons. The program uses both hard-coded expression obtained via analytical BCFW recursion, and numerical BCFW recursion. Using the latter, color-ordered amplitudes may be calculated to essentially arbitrary multiplicity. Squared amplitudes summed over colors and helicities are provided for up to six external gluons.

This paper continues as follows: in Section 2 the amplitudes that the program calculates are defined. Section 3 explains how color is treated. Section 4 describes the usage of the program, and Section 5 introduces the program with which it was validated. Section 6, finally, contains the summary.

2. Definitions

We consider the generally factorized formula for the gluonic contribution to a cross section in hadron collisions

$$\sigma\big(h_1(p_1)h_2(p_2)\to X\big)=\int d^4k_1\,F_1(k_1)\int d^4k_2\,F_2(k_2)\,\frac{\hat\sigma\big(g^*(k_1)g^*(k_2)\to X\big)}{4\sqrt{(k_1\cdot k_2)^2-k_1^2k_2^2}}. \tag{1}$$

This formula is very general, and g^* does not necessarily refer to an off-shell gluon. In collinear factorization, for example, we would have

$$F_i(k_i)=\frac{1}{2N_c}\int_0^1\frac{dx_i}{x_i}\,f_i(x_i,\mu)\,\delta^4(k_i-x_i\,p_i), \tag{2}$$

where f_i is the collinear pdf for a hadron of type i. We include the factors establishing averaging over spins and colors in F_i here. In the hybrid HEF [38], for example, F_2 would be as above, while F_1 would be given by

$$F_1(k_1)=\frac{1}{N_c}\int\frac{d^2k_T}{2\pi}\int_0^1\frac{dx_1}{x_1}\,\mathcal{F}_1(x_1,k_T,\mu)\,\delta^4(k_1-x_1\,p_1-k_T), \tag{3}$$

where $\mathcal{F}_1(x_1,k_T,\mu)$ is the unintegrated gluon density.

The symbol X stands for a partonic final state, for example a number of on-shell gluons. The partonic cross section $\hat\sigma$ is given by

$$\hat\sigma\big(g^*(k_1)g^*(k_2)\to X\big)=\int d\Phi(k_1,k_2\to X)\,\big|\mathcal{A}(g^*g^*\to X)\big|^2\,\mathcal{O}(X). \tag{4}$$

The phase space integration includes the summation over all color and spin degrees of freedom. The observable $\mathcal{O}$ turns the partonic final state into a physical final state, for example through a jet algorithm. This function also contains the necessary symmetry factors related to the final state. We concentrate on the amplitude $\mathcal{A}(g^*g^*\to X)$ from now on, for the case that X stands for a number of on-shell gluons.

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