Computer Physics Communications 189 (2015) 72–76

Contents lists available at ScienceDirect

Computer Physics Communications

journal homepage: www.elsevier.com/locate/cpc

Modeling gridlock at roundabout

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ARTICLE INFO

Article history:
Received 13 September 2014
Accepted 24 December 2014
Available online 2 January 2015

Keywords:
Cellular automata
Phase diagram
Traffic flow
Congestion

ABSTRACT

We propose a cellular automaton model to study the traffic patterns at a roundabout. We obtain the complete phase diagram which consists of four distinct phases: free flow, congestion, bottleneck, and gridlock. For the six possible transitions among these four phases, we observe only five transitions. Transition between congestion and gridlock is forbidden. Transition between free-flowing and gridlock is abrupt. Transition between bottleneck and gridlock is smooth. Instead of directly related to congestion, gridlock can be taken as an extreme limit of bottleneck. We show that both bottleneck and gridlock are caused by the traffic interweave, which can be well characterized by a new parameter χ. Theoretical implications are discussed. The cellular automata can serve as a basic model for practical applications.

1. Introduction

To understand the traffic congestion more thoroughly has been a challenge raised in the modern society. With conventional wisdom from personal perspectives, emergence of congestion is often blamed to some particular drivers who violate the traffic rules. A different viewpoint from system dynamics reveals that traffic congestion can be the result of system instability. Under certain conditions, congestion is unavoidable even if everyone follows the rules. In this work, we will focus on the most severe kind of congestion, i.e., gridlock. In contrast to the free flow, vehicles in the congestion are trapped in the stop-and-go cycles. The gridlock is often taken as an extreme case of congestion where the cycle time diverges. This work aims to clarify the relation between congestion and gridlock.

Traffic dynamics can be explored either by differential equations or cellular automaton rules. The former takes advantage of an analogy to macroscopic hydrodynamics or microscopic Newtonian dynamics [1]. The latter mimics the essence of our daily experiences [2,3]. To study the traffic flow on a straight roadway with homogeneous features, different approaches can be applied to reach more or less the same result. When the roadway becomes complicated, the complex formulation of differential equations is quite involved. On the other hand, the rule-based approach can be applied straightforwardly on any roadways. In this work, we propose a cellular automaton model to study the emergence of gridlock at a traffic roundabout.

In contrast to the highway traffic, urban traffic is characterized by the intersections. Traffic from different directions meet at an intersection. Sharing the limited space by vehicles causes numerous conflicts. There are basically two categories of schemes available to resolve the traffic conflicts. Schemes in the first category require a vehicle to come to a full stop. Vehicles take turn to access the intersection. Typical examples are stop sign and traffic signal [4–6]. Schemes in the second category aim to avoid the full stop of any vehicle. Traffic roundabout (rotary, circle) is the typical example [7–11]. Compared to traffic signal, an obvious advantage of a traffic roundabout is to keep traffic flowing without any vehicle being stopped. With naive expectations, such a goal is feasible if everyone follows the rules. However, there are also contrary opinions that traffic roundabout causes more congestion. As a roundabout involves many parameters, most previous works explored only a few special cases in the huge parameter space. In this work, we propose a cellular automaton model to have a complete analysis of the traffic flow around a traffic roundabout. We present the model in the next section, followed by the results of numerical simulations. Some analytical properties will also be discussed.

2. Model and simulation

The system consists of eight straight roadways connected to a ring [12], see Fig. 1. All the roadways including the ring are one-lane and one-way. Traffic direction on each roadway is indicated by the arrow. Incoming roadways are indexed by odd numbers; outgoing roadways are indexed by even numbers. Vehicles move into the system through odd-numbered boundaries. After traveling a certain distance in the ring, vehicles move out of the system through even-numbered boundaries. The total number of vehicles

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http://dx.doi.org/10.1016/j.cpc.2014.12.011
0010-4655/

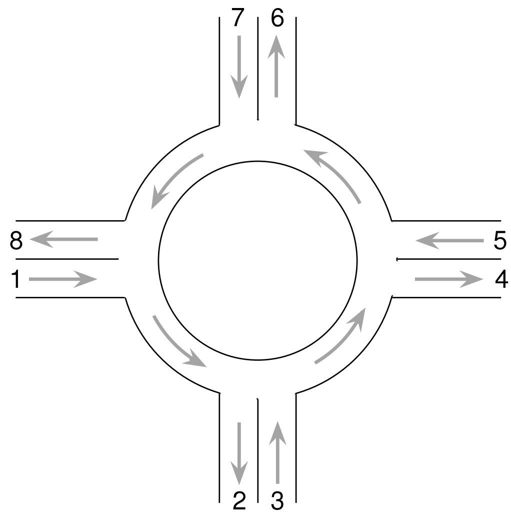

Fig. 1. System configuration of a roundabout.

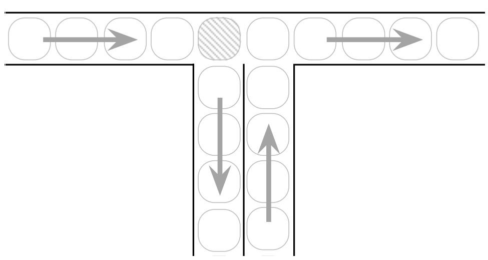

Fig. 2. Configuration of a T-shaped intersection.

in the system is not conserved. The roundabout can be taken as four connected T-shaped intersections [13], as shown in Fig. 2. The dynamics is governed by the Asymmetric Simple Exclusion Process (ASEP) with parallel update [14]. Roadways are divided into discrete cells. Each cell can be either empty or occupied by one vehicle. If an empty cell in front is available, vehicle moves forward in the next time step. Vehicle on each cell has an unique direction to follow. The only exception is the vehicle at the exit intersection, as shown by the shaded cell in Fig. 2. On the shaded cell, a vehicle may turn right to exit the ring, or move forward to remain in the ring. These two choices are determined by a quench variable assigned to each vehicle while entering the system. It is reasonable to assume that vehicles approaching the roundabout have their own predestined journeys. A traffic conflict can be expected at the entry intersection, i.e., the cell next to the shaded cell shown in Fig. 2. Without further regulations, two vehicles might move into that cell simultaneously. To avoid such a conflict, we adopt a conventional regulation that entering vehicle should yield to vehicle in the ring. The vehicle occupied the shaded cell has the right-of-way to move forward.

The dynamics is deterministic. While the boundary conditions are stochastic. At odd-numbered boundaries, if the first cell is empty, a new vehicle is added stochastically with a finite probability α_i. The destiny of this newly added vehicle is also selected stochastically. With a finite probability P_{ij}, the vehicle enters the system through boundary i and exits through boundary j, where $i = 1, 3, 5, 7$, and $j = 2, 4, 6, 8$. At even-numbered boundaries, vehicles leave the system with a finite probability β_j. Basically the injection α_i's control the inflow, the removal β_j's control the outflow, and the distribution P_{ij}'s control the traffic interweave in the ring. In this model, all vehicles follow the same rules. The typical characteristics of a traffic roundabout are preserved in the model as (a) traffic entering the ring must yield the right-of-way to traffic already in the ring; (b) no lane changes occur within the ring; (c) vehicle speeds are low.

In this model, the roadway configuration has the four-way symmetry. When the boundary conditions also respect this symmetry, the number of parameters can be reduced significantly [12],

$$\alpha_1 = \alpha_3 = \alpha_5 = \alpha_7 \equiv \gamma, \tag{1}$$

$$\beta_2 = \beta_4 = \beta_6 = \beta_8 \equiv \delta, \tag{2}$$

$$P_{12} = P_{34} = P_{56} = P_{78} \equiv Q_1, \tag{3}$$

$$P_{14} = P_{36} = P_{58} = P_{72} \equiv Q_2, \tag{4}$$

$$P_{16} = P_{38} = P_{52} = P_{74} \equiv Q_3, \tag{5}$$

$$P_{18} = P_{32} = P_{54} = P_{76} \equiv Q_4. \tag{6}$$

The traffic demand of the whole system is controlled by the injection γ and the removal δ in roadway boundaries. The traffic distribution in the ring is controlled by Q_1, Q_2, Q_3, Q_4, with a constraint $Q_1 + Q_2 + Q_3 + Q_4 = 1$, where Q_1 is the ratio of vehicles to travel a quarter of the ring and then to leave at the first exit; Q_2 is the ratio of vehicles to travel half of the ring and then leave at the second exit; Q_3 is the ratio of vehicles to travel three quarters of the ring and then leave at the third exit; Q_4 is the ratio of vehicles to travel the full ring to make a U-turn. As these parameters vary, the traffic system displays four distinct phases: free flow, congestion, bottleneck, and gridlock. In the free flow, all vehicles move freely. The incoming roadways and the outgoing roadways share the same vehicular density; while the density on the ring assumes a higher value owing to the traffic interweave. In the congestion, traffic jams appear in all the roadways. Again, the incoming roadways and the outgoing roadways share the same density. The density on the ring assumes a lower value to balance the flow. In the bottleneck phase, the traffic ring becomes a bottleneck and causes a density difference to build up between incoming and outgoing roadways. Vehicles will not be able to enter the ring smoothly. Traffic jams emerge in the ring and also in the incoming roadways. As a consequence, the incoming roadways have a higher vehicular density than the density on the ring; while the outgoing roadways remain free-flowing and have a lower density. In the gridlock, both the ring and the incoming roadways are jam-packed with vehicles. No further vehicles can enter the system. The outgoing roadways become empty. The characteristics of these four phases are summarized in Table 1, where ρ_i, ρ_o, and ρ_r denote respectively the densities of the incoming, the outgoing, and the ring roadways.

3. Bottleneck and gridlock

We observe that traffic flow in the bottleneck phase is mainly controlled by the interweave traffic in the ring, which can be conveniently summarized by a new parameter χ defined as

$$\chi \equiv Q_1 + 2Q_2 + 3Q_3 + 4Q_4 = 4 - 3Q_1 - 2Q_2 - Q_3, \tag{7}$$

where $1 \leq \chi \leq 4$. Basically $(\chi/4)$ denotes the average range of a vehicle traveling in the ring roadway. When $\chi = 1$, vehicles travel only a quarter of the ring and the traffic from different directions do not interweave with each other; when $\chi = 4$, vehicles travel the full ring and the traffic interweave most heavily. In Fig. 3, we plot the vehicular densities as functions of both the injection γ and the parameter χ, where the removal $\delta = 0.6$ is fixed. The four different phases can be easily discerned. At a small γ, traffic demand is small and can be well-satisfied. All vehicles move freely. When χ is small, the system is basically the same as a homogeneous roadway. Free-flowing transits to congestion as γ increases to larger than δ. At a larger χ, free-flowing transits to bottleneck as γ increases. At an even larger χ, free-flowing transits to gridlock as γ increases. It is interesting to note that the gridlock phase dominants the parameter space (γ, δ, χ), where $\gamma, \delta \in [0, 1]$ and $\chi \in [1, 4]$.