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# 3D particle tracking velocimetry using dynamic discrete tomography

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ABSTRACT

Particle tracking velocimetry in 3D is becoming an increasingly important imaging tool in the study of fluid dynamics and combustion as well as plasmas. We introduce a *dynamic discrete tomography* algorithm for reconstructing particle trajectories from projections. The algorithm is efficient for data from two projection directions and exact in the sense that it finds a solution consistent with the experimental data. Non-uniqueness of solutions can be detected and solutions can be tracked individually.


## 1. Introduction

Particle tracking velocimetry (PTV) is a diagnostic technique that plays an important role in studying flows [1–8] including combustion [9–16]. It has also been used to study plasma [17–24]. In PTV the motion of particles is followed in a sequence of images to measure their instantaneous velocities. In complex plasmas the particles themselves are the subject of interest [25–29] whereas in fluids the particle velocities are nearly the same as the local flow velocities which can hence be studied by PTV.

PTV is similar to the related particle image velocimetry (PIV) [2]. PTV tracks the motion of individual particles whereas PIV tracks the motion of groups of particles statistically. In PTV measurements the concentration of tracer particles is therefore significantly lower than in PIV measurements. In traditional *two-dimensional* PTV or PIV measurements, the flow field is illuminated by a thin laser sheet. Light is scattered from the tracer particles in this laser sheet and imaged on a CCD camera. From a series of images we can then obtain 2D flow velocities in the plane of the laser sheet [30,31]. In stereo PIV measurements the laser sheet is observed with two cameras, and velocities in the plane of the laser sheet can be obtained [32]. PIV techniques have been extended to volumetric 3D measurement by scanning planar PIV [33], holographic PIV [34], and tomographic PIV [35]. We study the 3D PTV mode of operation in which individual particles are tracked to obtain 3D velocity vectors in a measurement volume [33,36–38]. The particles either scatter light from a volumetric illumination of the measurement volume or they glow by themselves as often in plasma. 3D PTV [38] is advantageous if the density of particles is intrinsically low or has to be limited.

Current tomographic particle tracking methods are based on the multiplicative algebraic reconstruction technique (MART) [39] and its variants [36,40]. These are methods for reconstructing the distribution of multiple-pixel sized particles modeled as graylevel images. The graylevel can take any value and is a continuous quantity. The subsequent binarization is usually performed by comparison of the graylevel to a threshold. This procedure is not guaranteed to yield solutions that are consistent with the data. In contrast, our algorithm returns binary solutions that are consistent with the data as this is explicitly included as a constraint in the imaging model. Information from previously reconstructed frames is incorporated in the reconstruction procedure that is formulated as a *discrete optimization problem*. To our knowledge, discrete optimization methods have not previously been applied in PTV.

Existing PTV algorithms (such as [41–43]) rely on the following assumptions: (a) applied reconstruction routines are computationally efficient; (b) the reconstructions are stable, i.e., reconstruction errors are small whenever measurement errors are small; and (c) the reconstructions are uniquely determined by the data. The

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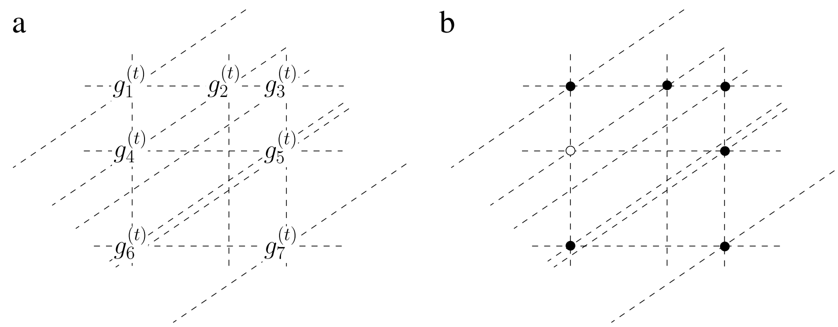


**Fig. 1.** A two-dimensional example for three projection directions (signal-recording lines indicated by dashed lines). (a) The grid $G^{(t)}$, and (b) a possible solution $\underline{\vec{x}}^{(t)}$ representing a set of particles that are consistent with the projections (black and white dots corresponding to $\xi_i^{(t)} = 1$ and $\xi_4^{(t)} = 0$, respectively).

algorithms are therefore generally not able to deal with ambiguities in the reconstruction and typically require heuristic knowledge for particle tracking. Many PTV algorithms, including the one presented in this paper, utilize information from previously reconstructed frames [44–47].

Here we discuss efficiency, stability, and uniqueness of the trajectory reconstructions in 3D PTV by relating them to results from the mathematical field of *discrete tomography*, which has originally been developed for reconstructing crystalline objects from high-resolution transmission electron microscopy (HRTEM) data [48]; see also [49–51]. Discrete tomography is preferred over conventional computer tomography (CT) in such tasks, because CT algorithms are, firstly, not well-suitable for reconstructing distributions of pixel-sized objects and, secondly, well-known to generate severe artifacts in cases where projection data is available only from a few directions.

We introduce a *dynamic* discrete tomography algorithm for 3D PTV, which can efficiently reconstruct trajectories of pixel-size objects from projection data acquired from two directions. The projections are assumed to be acquired along lines, i.e., two 1D detectors are required for particles that are confined to a plane (which could be also called 2D PTV) whereas two 2D detectors are required for particle tracking in 3D. Performing reconstructions from only a few projections can be important in experimental set-ups with limited optical access. For example, in machines for studying high-temperature plasmas the available space for diagnostics is usually very limited and possibilities of reducing the amount of in-vessel equipment are beneficial [23,24,52,53].

Another potential application of the 3D PTV algorithm is a recent experiment on a gliding arc [54,55]. A gliding arc is a thin string-like plasma column that is suspended between two electrodes while it is convected in a turbulent free jet [56–58]. The gliding arc can be used in surface treatment (adhesion) [59], bacterial inactivation [60], and many other applications. It has been found by PTV [58] and by measurements with a Pitot tube [57] that the jet flow is about 10%–20% faster than the plasma column. Spatial resolution of the slip velocity (i.e., the velocity of the jet flow measured relatively to the velocity of the plasma column) is not available in the literature as the seeding density of particles was too low. In the gliding arc experiment the density of seed particles should not be too high as the plasma column might otherwise be disturbed. Further, to study the gliding arc, images at a frame rate of 420 kHz and a resolution of $64 \times 128$ pixels have been used [54]. The pixel resolution was this low for the benefit of the high frame rate.

We introduce our imaging model in Section 2, present our dynamic discrete tomography algorithm for 3D PTV in Section 3, and discuss stability and uniqueness of the solutions in Section 4. Performance of the algorithm is demonstrated in Section 5, followed by the conclusions in Section 6.

## 2. Imaging model

We assume that one-dimensional projections of the particles are acquired from at least two projection directions (i.e., projections, either in 2D or 3D, are acquired along lines from at least two directions). The number of projection directions is henceforth denoted by $m$. In 3D PTV applications, a projection can be understood as a mapping from 3D space to 2D space, i.e. from real space to a photo image. Similarly, an analogous mapping from 2D space to 1D space can be considered if the sample is confined to a plane. The projection can be represented by binary-valued functions where 1 represents detection of a particle and 0 represents non-detection. We remark that this differs from PIV and computerized tomography cases in which intensities are measured that can take any value and that are therefore continuous quantities. In PTV applications, however, it is challenging to relate the detected brightness level to the number of particles lying on the corresponding projecting lines. The binary-valued data, on the other hand, are readily available.

A parallel beam geometry as indicated in some of the figures is not essential in our case. For $m$ projection directions, $m$ projecting lines pass through every particle. The intersections of these projecting lines for every projection direction are called *candidate points*. The set of candidate points is the so-called *(candidate) grid*; it contains the set of all particle positions and typically many additional points that are all other intersections of these projecting lines. We assume throughout the paper that we have $n$ particles. Hence we have at most $n$ projecting lines for each projection direction, and thus the number of grid points in the corresponding 2D grid does not exceed $n^2$. However, the grid can differ in different time steps since multiple particles might be lying on a projecting line. Also note that the grid can be computed efficiently from the data by computing the intersection points of the corresponding projecting lines.

We consider now the reconstruction problem at time $t$. To each point $g_i^{(t)}$ of the candidate grid $G^{(t)}$ containing $l(t)$ points we associate a variable $\xi_i^{(t)}$. Presence or absence of a particle at $g_i^{(t)}$ is indicated by the value $\xi_i^{(t)} = 1$ and $\xi_i^{(t)} = 0$, respectively; see also Fig. 1. The requirement that any solution $\underline{\vec{x}}^{(t)} := (\underline{\xi}_1^{(t)}, \dots, \underline{\xi}_{l(t)}^{(t)})^T \in \{0, 1\}^{l(t)}$ obtained by a reconstruction algorithm should be consistent with the projection data can be described by a *0-1-system of linear inequalities*:

$$A^{(t)}\vec{x}^{(t)} \geq \vec{b}^{(t)}, \quad \vec{x}^{(t)} \in \{0, 1\}^{l(t)}, \tag{1}$$

where $\vec{b}^{(t)} := (1, \dots, 1)^T \in \{1\}^{k(t)}$ represents the data; $k(t)$ denotes the total number of measurements, and $A^{(t)} \in \{0, 1\}^{k(t) \times l(t)}$ collects the individual variables' contributions to the signal as specified by the acquisition geometry (for the top horizontal line in Fig. 1, for instance, we would have $\xi_1^{(t)} + \xi_2^{(t)} + \xi_3^{(t)} \geq 1$). Note