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Hypersampling of pseudo-periodic signals by analytic phase projection

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A B S T R A C T

A method to upsample insufficiently sampled experimental time series of pseudo-periodic signals is proposed. The result is an estimate of the pseudo-periodic cycle underlying the signal. This "hypersampling" requires a sufficiently sampled reference signal that defines the pseudo-periodic dynamics. The time series and reference signal are combined by projecting the time series values to the analytic phase of the reference signal. The resulting estimate of the pseudo-periodic cycle has a considerably higher effective sampling rate than the time series. The procedure is applied to time series of MRI images of the human brain. As a result, the effective sampling rate could be increased by three orders of magnitude. This allows for capturing the waveforms of the very fast cerebral pulse waves traversing the brain. Hypersampling is numerically compared to the more commonly used retrospective gating. An outlook regarding EEG and optical recordings of brain activity as the reference signal is provided.

1. Introduction

Many signals in the biological and biomedical sciences are of a pseudo-periodic nature with irregularly spaced, stretched, or otherwise distorted variations of a repeating cycle. An example for a pseudo-periodic cycle is the characteristic QRS complex observed in electric recordings of the heart [1]. Another example are patterns of electrical activity of the brain observed in electroencephalographic (EEG) surface recordings [2]. Those signals usually can be measured with a sufficient sampling rate to resolve their underlying pseudo-periodic cycles (QRS-complex, EEG waveform, respectively). However, often it is not possible to measure the effects of the pseudo-periodic dynamics in parts of the body that cannot be accessed so easily, for example deep within the brain. The method of choice to obtain signals from anywhere in the brain is magnetic resonance imaging (MRI). Dynamic or functional MRI of the brain is typically sampled at an insufficient rate to resolve the cardiac cycle or EEG patterns [3]. In order to investigate the pseudo-periodic signal in a particular location within the brain, one solution is to upsample the MRI signal at that point with an effective sampling time that is much smaller than the average cardiac cycle or EEG waveform period. The cardiac or EEG recordings then can serve as a reference used to define the pseudo-periodicity of the dynamics of interest.

Here, an efficient upsampling procedure, called hypersampling, is described. Hypersampling consists of an upsampling of the undersampled time series by using the method of analytic phase projection (APP). Hypersampling can be seen as a generalization of retrospective gating [4,5]. Whereas in retrospective gating a recurring template is identified from the reference signal, in hypersampling the continuous phase underlying the pseudo-periodic reference signal is identified from the reference signal. This phase estimate is then used for APP.

The organization of this manuscript is as follows: First, hypersampling by APP is described in Section 2. Hypersampling is demonstrated on simulations in Section 3. In Section 4, these concepts are applied to an MRI of the brain in order to visualize the very fast pulse waves traversing the brain, which normally cannot be resolved with MRI. A discussion including a comparison with retrospective gating and possible further applications to other hybrid systems with fast and slow time scales concludes the manuscript. An appendix provides code for hypersampling via APP, and the supplementary data a video of the pulse waves observed in the human brain.

Supplementary video related to this article can be found at https://doi.org/10.1016/j.compbiomed.2018.05.008.

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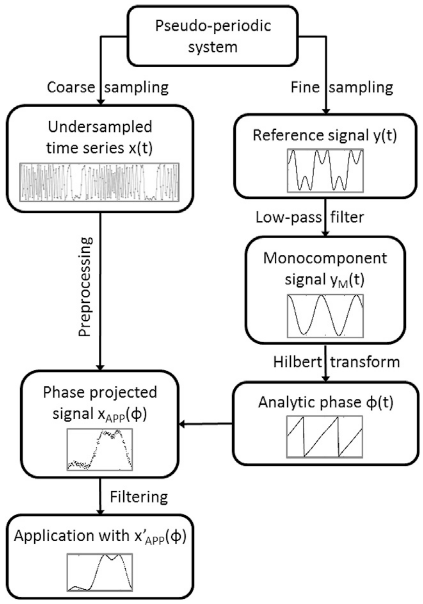

Fig. 1. Schematics of the analytic phase projection (APP) method to upsample an undersampled signal. Please refer to text for details.

2. Method

2.1. Overview

Hypersampling by analytic phase projection (APP) is summarized in Fig. 1.

Two signals are acquired from the system under study: An undersampled pseudo-periodic time series x(t) and a sufficiently sampled pseudo-periodic reference signal y(t). The reference signal and the time series are acquired during the same time interval and are assumed to have the same pseudo-periodicity. Then, a monocomponent signal $y_M(t)$ is obtained by low-pass filtering the reference signal y(t) (box "Monocomponent signal $y_M(t)$"). Monocomponent signals have a monotonically increasing phase. In other words, in a monocomponent signal the instantaneous frequency or time derivative of its phase is non-negative at any time [6]. This phase monotonicity is needed later on in the phase projection step, which requires the phase to be a piecewise invertible function. In practice, the phase of a signal can only be obtained modulo an interval of length 2π, which causes phase resets. At phase resets, the phase has a discontinuity from a value near π to a value near -π, thereby crossing the zero line. The phase is estimated from the monocomponent signal as its analytic phase (box "Analytic phase Φ(t)"; two phase resets are visible). The analytic phase is interpolated to the time series sampling times. Then, the time series values are assigned to their corresponding phase values (box "Phase projected signal $x_{APP}(\Phi)$"). In other words, a coordinate transformation from time to phase is performed: Whereas the original time series depends on time, the phase-projected time series depends on the pseudo-periodic cycle phase. The time series values themselves are not altered, they are just re-ordered, thus the description as a "projection". The method is referred to as "hypersampling" because the main requirement is that the underlying pseudo-periodic process is sampled over a time that spans as many pseudo-periods as possible. Finally, depending on the particular application, it might be necessary to further filter the result in order to obtain an estimate for the phase-projected cycle (box "Application with $x'_{APP}(\Phi)$").

2.2. Computational details

A monocomponent signal can be written as the product of an instantaneous amplitude $\rho(t) \geq 0$ and an instantaneous phase factor cos (Φ(t)), or as an amplitude-phase modulation [6,7]. Writing the signal $y_M(t)$ as an amplitude-phase modulation

$$y_M(t) = \rho(t)\cos\Phi(t) \,, \tag{1}$$

its analytic extension is

$$y_A(t) = y_M(t) + i y_H(t) = \rho(t)e^{i\Phi(t)} \,, \tag{2}$$

with the Hilbert transform [6]

$$y_H(t) = \frac{1}{\pi} P \int_R \frac{y_M(\tau)}{t-\tau} d\tau = \lim_{\varepsilon\to 0^+} \frac{1}{\pi} \int_{|t-\tau|>\varepsilon} \frac{y_M(\tau)}{t-\tau} d\tau \,. \tag{3}$$

The integral in this expression is a principal value integral. The analytic extension (2) expressed via the Hilbert transform (3) provides a unique expression for the amplitude-phase modulation (1), the "canonical amplitude-phase modulation" [6]. The Hilbert transform itself can be computed by standard signal processing software [8]. The analytic phase follows from the analytic signal as

$$\Phi(t) = \arg(y_A(t)) = \arg(y_M(t) + i\, y_H(t)) \,. \tag{4}$$

The argument function here is the four-quadrant inverse tangent relation, sometimes denoted $\mathrm{atan2}(y_H(t), y_M(t))$. Its principal values are restricted to the interval (-π, π]. In a monocomponent signal, the instantaneous frequency is always non-negative, i.e., $d\Phi(t)/dt \geq 0$, for all time points where it is defined. Thus, the analytic phase is monotonically increasing, and decreasing only during phase resets. The phase monotonicity can be checked, for example, visually by graphing the analytic phase. If necessary, the low-pass filter can be adjusted such as to improve monotonicity of the analytic phase. Depending on the application, it might also be necessary to preprocess the time series, for example to remove trends. Once an approximately monotonic phase of the reference signal has been obtained, one can proceed with the APP, which combines the time series and the analytic phase of the reference signal:

The (preprocessed) pseudo-periodic time series x(t) is sampled at times t_i. The sampling times of the analytic phase Φ(t) are denoted by τ_j. The analytic phase projection is a coordinate transformation of the time series sampling times to the analytic phase,

$$\mathrm{APP}: \quad x(t_i) \to x_{APP}(\Phi_i) \,. \tag{5}$$

The index i assumes values from 1 to N, the number of time series samples. Here, $\Phi_i = \Phi(t_i)$ is the analytic phase Φ(t) numerically interpolated to the signal sampling time t_i. This interpolation should be quite accurate in general, as the analytic phase of a monocomponent signal is an approximately smooth function, if sufficiently sampled, except at phase resetting points. For phases near phase reset, the interpolation can become inaccurate and it might be necessary to provide corrective measures, for example discarding outliers. If to each time series sampling time t_i there is a corresponding reference signal sampling time $\tau_j = t_i$, the interpolation step can be omitted, i.e., the phases $\Phi(\tau_j)$ are taken directly as $\Phi_i = \Phi(\tau_j = t_i)$.