Computers in Biology and Medicine 96 (2018) 106–115

Contents lists available at ScienceDirect

Computers in Biology and Medicine

journal homepage: www.elsevier.com/locate/compbiomed

A vectorized Levenberg-Marquardt model fitting algorithm for efficient post-processing of cardiac T_1 mapping MRI

Shufang Liu [a,b,c,*], Aurelien Bustin [a,b,c], Pauline Ferry [c,d], Andrei Codreanu [e], Darius Burschka [a], Anne Menini [b], Freddy Odille [c,d,f]

[a] Technische Universität München, Department of Computer Science, Munich, Germany
[b] GE Global Research, Munich, Germany
[c] Imagerie Adaptative Diagnostique et Interventionnelle, Université de Lorraine, Nancy, France
[d] IADI, INSERM, Nancy, France
[e] Centre Hospitalier de Luxembourg, Luxembourg
[f] CIC-IT 1433, CHRU de Nancy, Nancy, France

ARTICLE INFO

Keywords:
T_1 mapping
Curve fitting
Cardiac MR
Relaxometry
MR parameter mapping

ABSTRACT

Purpose: T_1 mapping is an emerging MRI research tool to assess diseased myocardial tissue. Recent research has been focusing on the image acquisition protocol and motion correction, yet little attention has been paid to the curve fitting algorithm.

Methods: After nonrigid registration of the image series, a vectorized Levenberg-Marquardt (LM) technique is proposed to improve the robustness of the curve fitting algorithm by allowing spatial regularization of the parametric maps. In addition, a region-based initialization is proposed to improve the initial guess of the T_1 value. The algorithm was validated with cardiac T_1 mapping data from 16 volunteers acquired with saturation-recovery (SR) and inversion-recovery (IR) techniques at 3T, both pre- and post-injection of a contrast agent. Signal models of T_1 relaxation with 2 and 3 parameters were tested.

Results: The vectorized LM fitting showed good agreement with its pixel-wise version but allowed reduced calculation time (60 s against 696 s on average in Matlab with $256 \times 256 \times 8(11)$ images). Increasing the spatial regularization parameter led to noise reduction and improved precision of T_1 values in SR sequences. The region-based initialization was particularly useful in IR data to reduce the variability of the blood T_1.

Conclusions: We have proposed a vectorized curve fitting algorithm allowing spatial regularization, which could improve the robustness of the curve fitting, especially for myocardial T_1 mapping with SR sequences.

1. Introduction

Many myocardial diseases are accompanied by the excessive deposition of myocardial collagen, which results in the change of myocardial structure [1,2]. Thanks to the fast development of cardiac magnetic resonance imaging, the change of the myocardial structure can be evaluated noninvasively by measuring the pixel-wise longitudinal relaxation time (T_1) of the heart tissue, before and/or after the injection of a Gadolinium-based contrast agent [3]. Cardiac T_1 maps are generally produced using either an inversion recovery (IR) or a saturation recovery (SR) acquisition scheme, which means that several images need to be acquired sequentially after a variable delay following inversion (respectively saturation) of the magnetization, allowing a sampling of the T_1 relaxation curve for each pixel. IR techniques such as MOLLI [4] are most commonly used due to their larger dynamic range and better precision and reproducibility, whereas SR techniques such as SASHA or SMART1Map [5] provide more accurate T_1 values [6]. For both sequences, typical series of 8–11 images with different inversion/saturation times are acquired, and an exponential model fitting (using a 2-parameter or 3-parameter model) is performed for each pixel by nonlinear least-squares optimization [7].

A lot of work has been done investigating the acquisition sequence and motion correction techniques [8–11]. Yet the curve fitting algorithm has not been completely optimized. The accuracy and precision of the fitting process depend on many factors including the number of measurements (i.e. number of input images), the number of parameters

* Corresponding author. Institut für Informatik VI, Technische Universität München, Parkring 13, 85748, Garching bei München, Germany.
E-mail address: shufang.liu@tum.de (S. Liu).

https://doi.org/10.1016/j.compbiomed.2018.03.009
Received 31 July 2017; Received in revised form 6 March 2018; Accepted 11 March 2018

chosen for the fitting model (2 or 3), the algorithm used for nonlinear optimization, the choice of a good initial guess of the model parameters, and potentially the presence of artefacts and misregistration between the images. Fitting the recovery curve from magnitude images is a well known difficulty in IR sequences since the phase of the complex MR signal is lost during the magnitude reconstruction. The multi-fit algorithm has been proposed in Ref. [12] to deal with that issue, however it is relatively inefficient because multiple fits have to be tested. Alternatively, phase-sensitive reconstruction can be used to restore the signal polarity in the images [13]. However the phase data are not always clinically available. This is because reconstructing the phase accurately is a difficult problem is MRI, due to the need for accurate calibration of the phase of the transmitting and receiving coils (typically 8 to 30+ receiver channels in cardiac MRI), and/or for good coil combination algorithms which are still a research topic [14,15]. Using a FLASH readout and Bloch Equation simulations has been proposed in order to improve the signal evolution model [16]. An inversion group fitting algorithm has also been proposed to better model the inversion recovery behavior [17].

Model based image reconstruction has been investigated to accelerate the parametric mapping. Compressed sensing has been used to reconstruct the parametric map using overcomplete dictionaries [18]. Model based methods can also be used to estimate the T_2 maps and spin-density maps from the raw data [19]. Regularization has also been proposed for reducing noise in the parametric map reconstruction [20]. However, all these methods were applied to the raw image data and required time-consuming reconstruction methods. In the image domain, Poot et al. proposed an elegant framework for simultaneous estimation of parametric maps and noise maps with spatially smooth noise levels, based on maximum likelihood or maximum a posteriori estimation [21]. This approach was shown to improve the precision of quantitative parameter estimation in diffusion tensor MRI. In this work, we propose an alternative approach to optimize the model fitting process by reformulating the problem as a joint optimization whereby the T_1 models of all pixels in the image are solved simultaneously, using a matrix formulation. We hypothesize that this matrix formulation has two benefits: (i) it provides a vectorized version of the Levenberg-Marquardt algorithm which is more computationally efficient than the standard pixel-wise approach and is particularly well suited for modern architectures such as vector CPUs or graphics processor units (GPUs); (ii) additional constraints, such as spatial regularization, can be incorporated into the optimization to stabilize the fitting process and/or speed up convergence.

In this paper we first describe the proposed vectorized Levenberg-Marquardt algorithm and its exemplary implementation in the case of SR and IR T_1 mapping data with a 2- or 3-parameter model. Special care is given to the initialization: an automatic image segmentation technique is used as a preprocessing step to compute region-wise estimates of the model parameters (including T_1), which are used as initial guesses for the final vectorized T_1 fitting procedure. Motion is a well-known problem in cardiac imaging and non-rigid registration techniques have been applied recently to cardiac T_1 and T_2 mapping [10,11,22]. In this paper we use a home-made non-rigid registration to correct for breathing motion. Then the proposed vectorized approach is validated in terms of computational efficiency and stability of the model fitting.

2. Theory

2.1. Background on Levenberg-Marquardt optimization

We first consider a single model fitting problem associated with one pixel in the image. This can be treated as a least-squares minimization of the error between the acquired data y, a vector of N_m measurements ($N_m = 8\ to\ 11$ in this work), and the unknown model defined as a function f of an unknown parameter set p, a vector of a N_p elements ($N_p = 2\ or\ 3$ here):

$$\min_p ||f(p) - y||^2. \tag{1}$$

Levenberg-Marquardt algorithm (LM) [23–25] is a popular choice for solving Eq (1) when the function $f(p)$is nonlinear. Starting from an initial guess p of the parameters, the method consists of searching for an optimal refinement of the parameters δp, by linearizing the cost function around the current estimate:

$$\min_{\delta p} ||f(p+\delta p) - y||^2 \approx \min_{\delta p} ||J(p)\delta p - (y - f(p))||^2, \tag{2}$$

Where $J(p)$ is the Jacobian matrix of f with respect to the parameters, evaluated at the current guess p. $J(p)$ is of size $N_m \times N_p$. Therefore, LM involves solving a sequence of linear least squares problems using a regularized inversion of the Jacobian matrix in order to calculate the following update of the solution at a given iteration k:

$$\delta p = p_{k+1} - p_k = \left(J(p_k)^T J(p_k) + \lambda_k Id\right)^{-1} J(p_k)^T (y - f(p_k)), \tag{3}$$

Where Id is the identity matrix and λ_k is the LM regularization coefficient which is adapted throughout iterations. The rationale of LM is to start with a large value of λ_k, so the method behaves like a steepest gradient descent in the first iterations, then to decrease λ_k as p approaches the solution, so the method behaves like a quasi-Newton method in the last iterations. Such schemes–i.e. combining gradient descent and quasi-Newton - are thought to yield optimal convergence speed in the nonlinear optimization literature. Several variations of the LM technique have been proposed depending on the choice of λ_0, update rule for λ_k and stopping condition. Here we choose the following update rule [25,26]:

$$\begin{cases} if\ \Delta(p_k) > \varepsilon,\ set\ p_{k+1} = p_k + \delta p,\ \lambda_{k+1} = \min(\lambda_k \times 2,\ 10^7) \\ if\ \Delta(p_k) \le \varepsilon,\ set\ p_{k+1} = p_k \quad,\ \lambda_{k+1} = \max(\lambda_k/2,\ 10^{-7}) \end{cases}$$
$$with\ \Delta(p) = \frac{||f(p)||^2 - ||f(p+\delta p)||^2}{||f(p)||^2 - ||f(p) + J(p)\delta p||^2}, \tag{4}$$

and iterations are stopped when $||p_{k+1} - p_k||/||p_{k+1}|| < \tau$, with τ a given tolerance, or when a maximal number of iterations was reached.

2.2. Vectorized Levenberg-Marquardt formulation

In computer programming, vectorization consists of redesigning algorithms so that the same operations performed multiple times on different data are grouped into a single operation performed once on a large array of data. This generally results in improved performances as modern computer architecture (CPU or GPU) can make the most of these vector operations. At a higher level, vectorizing an algorithm processing pixels of an image can be thought of as processing them jointly. It is therefore possible to incorporate constraints such as spatial smoothness to improve the processing itself.

In order to formulate the vectorized version of LM for an image of N_{pix} pixels, we use the same framework as described in the previous section. However we redefine y to be the whole acquired image dataset, a vector of $N_{pix}N_m$ elements, p the concatenated parameter maps, a vector of $N_{pix}N_p$ elements, and f the fitting model function operating on images. Adding a spatial smoothness constraint leads to a vectorized version of Eq. (1):

$$\min_p ||f(p) - y||^2 + \mu||Gp||^2, \tag{5}$$

where μ is a scalar controlling the spatial regularization weight and G is an operator returning a concatenation of the spatial gradients of each parameter map, computed by forward differences. G is a sparse matrix of size $N_{dims}N_{pix}N_p \times N_{pix}N_p$, with N_{dims} the number of dimensions in the image (here $N_{dims} = 2$). The vectorized LM update formula becomes: