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QR-decomposition based SENSE reconstruction using parallel architecture

Irfan Ullah *, Habab Nisar, Haseeb Raza, Malik Qasim, Omair Inam, Hammad Omer

Department of Electrical Engineering, COMSATS Institute of Information Technology, Islamabad, Pakistan

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A B S T R A C T

Magnetic Resonance Imaging (MRI) is a powerful medical imaging technique that provides essential clinical information about the human body. One major limitation of MRI is its long scan time. Implementation of advance MRI algorithms on a parallel architecture (to exploit inherent parallelism) has a great potential to reduce the scan time. Sensitivity Encoding (SENSE) is a Parallel Magnetic Resonance Imaging (pMRI) algorithm that utilizes receiver coil sensitivities to reconstruct MR images from the acquired under-sampled *k*-space data. At the heart of SENSE lies inversion of a rectangular encoding matrix. This work presents a novel implementation of GPU based SENSE algorithm, which employs QR decomposition for the inversion of the rectangular encoding matrix. For a fair comparison, the performance of the proposed GPU based SENSE reconstruction is evaluated against single and multicore CPU using openMP. Several experiments against various acceleration factors (AFs) are performed using multichannel (8, 12 and 30) phantom and in-vivo human head and cardiac datasets. Experimental results show that GPU significantly reduces the computation time of SENSE reconstruction as compared to multi-core CPU (approximately 12x speedup) and single-core CPU (approximately 53x speedup) without any degradation in the quality of the reconstructed images.

1. Introduction

Magnetic Resonance Imaging (MRI) is a non-invasive and non-ionizing medical imaging technique that provides essential clinical information about the anatomy and functions of the human body. One major limitation of MRI is its long data acquisition time. Parallel Imaging is used to decrease the MRI scan time. Parallel Magnetic Resonance Imaging (pMRI) utilizes multiple receiver coils and under-sampled trajectories to significantly improve the scan time [1].

Sensitivity Encoding (SENSE) [2] is a pMRI technique that utilizes multiple receiver coil sensitivities to reconstruct MR images from the acquired under-sampled *k*-space data. SENSE is a valuable complement to gradient encoding that reduces the MRI scan-time. SENSE may help to relax the breath-holding requirements in cardiac imaging due to the reduction in scan time. It also opens ways for real time cardiac imaging without ECG triggering [3]. Furthermore, SENSE can be used to improve spatial resolution of the MR image. SENSE is available in clinical MRI scanners with a slight modification in implementation by different manufacturers e.g. Siemens (mSENSE), Philips (SENSE), Toshiba (SPEEDER), General Electric (ASSET) etc.

The most computation expensive part in SENSE is the matrix inversion operation which needs to be performed many times. The number of matrix inversion operations in SENSE depends on the number of receiver coils, Acceleration Factor (AF) which define the size of the encoding matrix [2]. Since the number of receiver coils has to be greater than the AF for SENSE, therefore the encoding matrix is rectangular [2]. Direct inversion methods [23] are used to invert a large rectangular matrix e.g. Cholesky factorization [3], left inverse method [4], QR decomposition [5] etc can be used for the inversion of the rectangular encoding matrix. Larger the size of the encoding matrix, the higher the computation time required for its inversion (as the number of equations and unknown variables increase) [23].

GPUs are a recent trend in the field of MRI and have been shown to successfully reduce the image reconstruction time by exploiting inherent parallelism of advance MRI reconstruction algorithms [3,4,6–9,21,22]. Nvidia provides Compute Unified Device Architecture (CUDA) framework for parallel computation and C/C++ language based programming interface to utilize GPU as a parallel programing resource. Utilizing GPU in the field of MRI to decrease computation time has shown significant improvement (10x ∼ 200x) in MR image reconstruction time [3–9,22] without any degradation in the quality of the reconstructed images.

In this work, GPU based SENSE reconstruction is proposed to exploit the inherent parallelism in the conventional SENSE algorithm. The proposed GPU based SENSE methodology employs parallel implementation

* Corresponding Author. Room 402 Cubator 1ne, Medical Image Processing Research Group Lab, Department of Electrical Engineering, COMSATS Institute of Information Technology, Islamabad, Pakistan.

E-mail address: irfan_ullah@comsats.edu.pk (I. Ullah).

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of QR decomposition to speed up the reconstruction process. Simulations are performed on the phantom, cardiac and in-vivo human head datasets. The performance of the proposed method is analyzed by comparing the results of the CPU and GPU reconstructions of SENSE (in terms of reconstruction time and Artifact Power(AP)) for different Acceleration Factors(AF).

2. Theory

2.1. SENSE reconstruction

SENSE algorithm requires receiver coil sensitivity maps and the under-sampled Cartesian data as inputs to reconstruct the fully sampled MR image [2]. In SENSE, good reconstruction results require an accurate estimation of the sensitivity profiles. The main equation of SENSE [2,17] is:

$$\vec{I} = \mathbf{C} \times \vec{\rho} \tag{1}$$

Where $\vec{I}$ represents the aliased pixels; **C** is the encoding matrix and $\vec{\rho}$ is the required solution image. Number of rows in the sensitivity encoding matrix represent the total number of receiver coils while the number of columns are defined by the Acceleration Factor (AF) [1]. A pictorial representation of equation (1) is shown in Fig. 1.

For equation (1), the equivalent matrix representation (for four receiver coils having AF = 4) would be:

$$\begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3 \\ \vec{I}_4 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix} \cdot \begin{bmatrix} \vec{\rho}_1 \\ \vec{\rho}_2 \\ \vec{\rho}_3 \\ \vec{\rho}_4 \end{bmatrix}$$

To find the solution image, the encoding matrix(**C**) needs to be inverted as given below:

$$\vec{\rho} = \mathbf{C}^{-1}\vec{I} \tag{2}$$

It is to be noted that normally **C** matrix is not a square matrix because AF has to be less than the number of receiver coils [1]. This constraint forms a rectangular encoding matrix and direct inversion methods that decompose the original matrix [3–5] into multiple matrices (for inversion) may be used for the inversion of the encoding matrix.

2.2. QR decomposition

QR-decomposition calculates the inverse of a rectangular matrix. The advantage of using QR decomposition algorithm is that it offers arithmetic operation level parallelism [13]. Hence, QR algorithm based applications are a suitable candidate for parallel architectures such as GPU to decrease the computation time. In QR-decomposition, a matrix **A** is decomposed into a product of matrices **Q** and **R** [14,15]:

$$\mathrm{A} = \mathrm{Q} \times \mathrm{R} \tag{3}$$

Where **Q** is an orthogonal matrix and **R** is an upper triangular matrix [13]. In this work, QR-decomposition is computed using classical Gram-Schmidt process. The classical Gram-Schmidt method decomposes a matrix **C** into matrices **Q** and **R** where **C** is a m-by-n rectangular matrix, **Q** is m-by-n orthogonal matrix and **R** is n-by-n upper triangular matrix [14,15].

Table 1 shows the pseudo-code for the classical Gram-Schmidt algorithm [13] to decompose a matrix **A** into sub-matrices **Q** and R. Here, Matrix **A** is a rectangular matrix of m x n dimensions and consists of independent vectors:

$$\mathbf{A} = \{ \vec{x}_1\ \vec{x}_2, \ldots, \vec{x}_k \} \tag{4}$$

where $\vec{x}_k$ denotes the kth column of matrix **A**. Classical Gram-Schmidt algorithm uses an iterative approach to find matrix **Q** that consists of n number of iterations and in each iteration one column of matrix **A** is used to produce one column of matrix **Q**. The first step to find matrix **Q** is to find the bases that consist of orthonormal vectors $\{\vec{z}_1, \vec{z}_2, \ldots, \vec{z}_k\}$. Therefore, vector $\vec{x_1}$ *to* $\vec{x_k}$ are used to compute the projections using equation (5):

$$\vec{r}_{ik}\vec{q}_1 = \text{where } \vec{r}_{ik} := (\vec{q}_i)^{\boldsymbol{H}}\vec{x}_{\boldsymbol{k}} \text{ and } \vec{q}_i \text{ is the ith column of matrix } \mathbf{Q}. \tag{5}$$

H in equation (5) denotes the conjugate transpose. The orthonormal vector $\vec{z_k}$ is calculated using equation (6):

$$\vec{z}_k = \vec{x}_k - \sum_{i=1}^{k-1} \vec{r}_{ik}\vec{q}_1 \tag{6}$$

The matrix **Q** is formed (equation (7) by concatenating the orthonormal transforms, all while obeying the invariants $\mathbf{A} = \mathbf{QR}$ and $\mathbf{Q^H Q} = \mathbf{I}$.

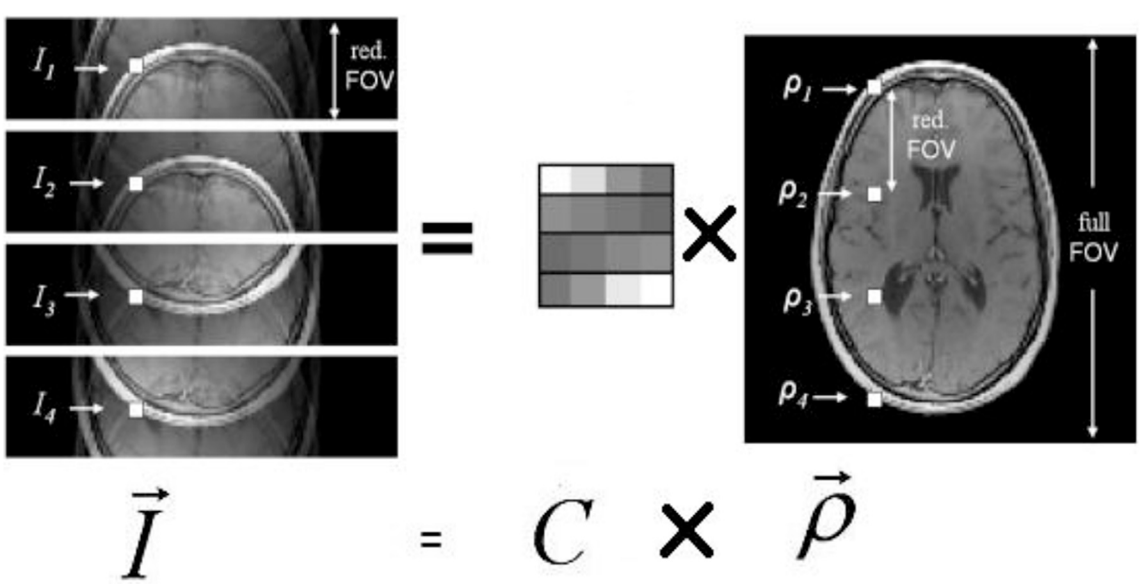

Fig. 1. Matrix notation of SENSE reconstruction.