Computers in Biology and Medicine 93 (2018) 127–138

Contents lists available at ScienceDirect

Computers in Biology and Medicine

journal homepage: www.elsevier.com/locate/compbiomed

Impeller-pump model derived from conservation laws applied to the simulation of the cardiovascular system coupled to heart-assist pumps

Yubing Shi [*,1], Theodosios Korakianitis [2]

School of Engineering, Queen Mary, University of London, UK

ARTICLE INFO

Keywords:
Ventricular assist device
Impeller pump
Cardiovascular dynamics
Mathematical model
Pressure difference-flow rate characteristics

ABSTRACT

Previous numerical models of impeller pumps for ventricular assist devices utilize curve-fitted polynomials to simulate experimentally-obtained pressure difference versus flow rate characteristics of the pumps, with pump rotational speed as a parameter. In this paper the numerical model for the pump pressure difference versus flow rate characteristics is obtained by analytic derivation. The mass, energy and angular momentum conservation laws are applied to the working fluid passing through the impeller geometry and coupled with the turbomachine's velocity diagram. This results in the construction of a pressure difference versus flow rate characteristic for the specific pump geometry, with pump rotational speed as parameter. Overall this model allows modifications of the pump geometry, so that the pump avoids undesirable operating conditions, such as regurgitant flow. The HeartMate III centrifugal pump is used as an example to demonstrate the application of the technique. The parameterised numerical model for HeartMate III derived by this technique is coupled with a numerical model for the human cardiovascular system, and the combination is used to investigate the cardiovascular response under different conditions of impeller pump support. Conditions resulting in regurgitant pump flow, the pump resulting in aortic valve closure and taking over completely the pumping action from the diseased heart, and inner ventricular wall suction at pump inlet are predicted by the model. The simulation results suggest that for normal HeartMate III operation the pump speed should be maintained between $3,100$ and $4,500\,rpm$ to avoid regurgitant pump flow and ventricular suction. To obtain optimal overall cardiovascular system plus pump response, the pump operating speed should be $3,800\,rpm$.

1. Introduction

Impeller pumps have been used in the last few decades in ventricular assist devices (VADs), which are designed to support the native diseased heart. Axial and centrifugal impeller pumps have been selected due to their simple mechanical structure as the preferred configuration for the new generation of VADs by several mainstream manufactures, including Abiomed Inc., Jarvik Heart Inc., Abbott Laboratories, Medtronic plc, ReliantHeart Inc. and others. Extensive numerical modelling and experimental studies have been conducted to create an overall understanding of the assisting action of impeller pumps on the native cardiovascular system dynamics. Computer modelling is the most widespread research technique due to its proven capability, efficiency and minimised cost in comparison to regulated in-vitro studies with blood, or in-vivo studies in animals and humans. In studying the characteristics of rotary type blood pumps, Smith et al [1] identified the trends in shut off pressure coefficient and non-dimensional characteristics of the pumps (i.e. non-dimensional functions of pressure difference, flow rate, impeller rotational speed, impeller diameter and working-fluid density). Non-dimensional pump characteristics were specifically analysed for centrifugal and axial impellers, both numerically and experimentally, including haemolysis effects in Refs. [2–4]. For numerical pump modelling, a number of computer models of varying complexity have been proposed and utilised to simulate the supporting effect of impeller pumps on cardiovascular system response. The simplest models describe the pump flow with a prescribed function. For example, Mitsui et al [5] modelled the pump using a pump flow equation of the format $Q = A^{*}(0.4 + 0.6{*}|sin(3\pi t/(2T))|)$, with A being a scaling factor, and T being the heart period. Such models prescribe the supporting action of the impeller pump, but are unable to reveal the interaction between the

* Corresponding author.
E-mail addresses: yubing.shi@northampton.ac.uk (Y. Shi), korakianitis@alum.mit.edu (T. Korakianitis).
[1] Present address: Faculty of Arts, Science and Technology, University of Northampton, UK.
[2] Present address: Parks College of Engineering, Aviation and Technology, Saint Louis University, St. Louis MO 63103, USA.

https://doi.org/10.1016/j.compbiomed.2017.12.012
Received 26 June 2017; Received in revised form 14 December 2017; Accepted 16 December 2017

Nomenclature

A	sectional area
c	absolute velocity
C	compliance effect of the vessel
CQ	flow coefficient
DT	time step
E	elastance of the heart chamber
EDV	end diastolic volume of the ventricular chamber
ESV	end systolic volume of the ventricular chamber
K, k	coefficient
L	inertial effect of the blood flow
m	mass
P	pressure
Q	flow rate
R	radius; flow resistance effect
rpm	revolutions per minute
t	time
T	heart period; torque
U	blade velocity
V	volume
w	relative velocity
W	power
Z	Number of blades
β	pump blade angle, i.e., the angle between the relative velocity and the meridional direction
θ	rotating angle of valve leaflet
ρ	density
σ	slip factor
ω	angular velocity

Subscripts

0	initial value; offset value; value for unstressed condition
a, b, c, Q	different parameters
ac	actual value
ao	aortic valve
ip	impeller pump
la	left atrium
lv	left ventricle
lvf	left ventricular failure
m	meridional
$mean$	mean value
mi	mitral valve
o	outlet
par	pulmonary arterioles
pas	pulmonary artery sinus
pat	pulmonary artery
pcp	pulmonary capillary
po	pulmonary valve
pvn	pulmonary vein
pwb	beginning of P wave
pww	duration of P wave
r	a specified intermediate instant in early systole
ra	right atrium
rv	right ventricle
s	shaft; systolic phase
$s1$	peak of systolic phase
$s2$	end of systolic phase
sar	systemic arterioles
sas	systemic aortic sinus
sat	systemic artery
scp	systemic capillary
svn	systemic vein
t	tangential
th	theoretical value
ti	tricuspid valve
twb	beginning of T wave
twe	end of T wave
tww	duration of T wave
vpi	value for pump inlet
vpo	value for pump outlet

impeller pump and the native cardiovascular system. More sophisticated models use polynomial equations to relate the variables of pump flow (Q), pressure difference across the pump (ΔP) and the pump rotating speed (ω), in the form of either $\Delta P = f(Q, \omega)$ or $Q = f(\Delta P, \omega)$. These equations model the experimentally measured pump characteristics (i.e., the pressure-flow curves under different pump speeds [6–14]). For example, Casas et al. [7] modelled the characteristics of a centrifugal pump using the equation $Q = a + bN + cDP + dN^2 + eDP^2 + fNDP$, where N is the pump speed in rpm, DP is the pressure difference across the pump, and a, b, c, d, e, f are coefficients. Waters et al [14] modelled the characteristics of a magnetic bearing supporting a continuous flow pump using the equation $Q = K_p \cdot DP + K_\omega \cdot \omega$, in which K_p and K_ω are empirical coefficients. These equations generally reveal the hydraulic dynamics in the impeller pump. In addition to the hydraulic effect, the mechanical effect related to pump rotor dynamics was either neglected [6–8,11–13] or modelled by considering the various moments acting on the rotor [9,10,14].

Since hemodynamic response is the major aim of such studies, accurate description of the $\Delta P = f(Q, \omega)$ or $Q = f(\Delta P, \omega)$ relations directly influences the validity of the modelling procedure. Among the above mentioned works, the polynomials constructed to describe the $\Delta P = f(Q, \omega)$ or $Q = f(\Delta P, \omega)$ relation were all chosen with the aim of optimally matching the pump characteristics data, and none of them explained why the equation chosen was the best match compared to others.

All physical systems and procedures follow physical laws, including the cardiovascular system hemodynamic response to heart-assist pumps. It would be useful if the selected numerical model for the impeller pump is derived from the governing physical laws for pump hydraulic effects. This would give the selected numerical equation relevance to engineering choices in the geometric design of the pump, and also assist in the derivation of additional useful information to aid the pump design and geometric optimisation. With this in mind, the current study explores analytical modelling of the impeller pump characteristics based on mass conservation, momentum conservation, energy conservation, and the velocity diagram of the working fluid as it passes through the impeller geometry. The outcome is a physics-based framework to assist the selection of the pump geometry, and also derives a numerical equation modelling the pump characteristics. The suggested equation is then parameterised and calibrated using published data [15] for an impeller pump. The quantified pump model is finally integrated into a model simulating human cardiovascular system response under the heart failure condition assisted with the impeller pump. HeartMate III is used as an illustrative example of application of the technique.

2. Materials and methods

The overall numerical model used in this study includes two parts: one for the native human cardiovascular system; and the other for the heart-assist pump, designated as VAD. Fig. 1 illustrates the block diagram of the whole system model. The impeller pump (VAD) is installed in parallel flow configuration with the flow provided by the native diseased