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A discrete particle model reproducing collective dynamics of a bee swarm

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ABSTRACT

In this article, we present a microscopic discrete mathematical model describing collective dynamics of a bee swarm. More specifically, each bee is set to move according to individual strategies and social interactions, the former involving the desire to reach a target destination, the latter accounting for repulsive/attractive stimuli and for alignment processes. The insects tend in fact to remain sufficiently close to the rest of the population, while avoiding collisions, and they are able to track and synchronize their movement to the flight of a given set of neighbors within their visual field. The resulting collective behavior of the bee cloud therefore emerges from non-local short/long-range interactions. Differently from similar approaches present in the literature, we here test different alignment mechanisms (i.e., based either on an Euclidean or on a topological neighborhood metric), which have an impact also on the other social components characterizing insect behavior. A series of numerical realizations then shows the phenomenology of the swarm (in terms of pattern configuration, collective productive movement, and flight synchronization) in different regions of the space of free model parameters (i.e., strength of attractive/repulsive forces, extension of the interaction regions). In this respect, constraints in the possible variations of such coefficients are here given both by reasonable empirical observations and by analytical results on some stability characteristics of the defined pairwise interaction kernels, which have to assure a realistic crystalline configuration of the swarm. An analysis of the effect of unconscious random fluctuations of bee dynamics is also provided.

1. Introduction

Self-organization and collective dynamics and behavior are ubiquitous phenomena in many biological and physical systems formed by populations of individuals, e.g., from colonies of bacteria to human crowds, see, for instance [9,32,44], and references therein. The description of swinging and coordinate movements of groups of animals, such as birds, fishes, insects, or certain mammals, has indeed increased in the last decades the multidisciplinary interest of various research communities, e.g., biologists, ecologists, sociologists, and applied mathematicians.

In this perspective, the theoretical and computational literature presents a wide range of approaches. For instance, in *discrete models*, characteristic of a *microscopic* point of view, the interacting individuals are described by localized particles, which move according to individual rules involving their position and velocity (or acceleration). The evolution of the overall system is indeed defined by proper sets of (first or second order) ordinary differential equations (ODEs). In this respect, discrete approaches are able to provide a detailed description of the dynamics of each agent and therefore represent a natural tool to investigate animal world-related collective phenomena (see, for instance [10,16,17,23,26,28,31,36,38]).

However, when the number of component particles is significantly large, as in the case of fishes [37] or myxobacteria [29,33], the solution of the resulting system of ODEs becomes computationally expensive and therefore different methods are needed. In this respect, *continuous models*, characteristic of a *macroscopic* point of view, rely on the definition of a proper density of agents, which evolves following (typically nonlinear) partial differential equations (PDEs). They are indeed based on conservation laws and phenomenological assumptions for their closure, see for example [3,45–47]. It is useful to remark that the rules of motion underlying discrete and continuous approaches often coincide: however, in the former case, they are related to the behavior of the single agents, whereas, in the second case, they account for the dynamics of the overall population density.

A bridge between the microscopic word and the macroscopic representation of the system is represented by *kinetic* models. Characteristics of a *mesoscopic* point of view, they are able to derive, employing hydrodynamic arguments, Boltzmann-like evolution laws for statistical distribution functions which describe the position and velocity of the

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components of the population of interest [5,7,34].

Some of the previous-cited mathematical approaches deal with the collective behavior of bee swarms, which represent an interesting problem to be studied. Such insect populations, which are typically composed by the old queen and by 10000–30000 worker individuals, in fact undergo a synchronized flight when they have the specific purpose of reaching the new nest site [42]. All colonies are subjected to this natural phenomenon, and every year beekeepers have to deal with it in late spring and early summer. In this period, as the weather warms up and flowers begin to bloom, the colony is in fact at the peak of its capacity and ready to produce a new hive. Entering in more details, when the migrating bees leave the original hive they first temporarily settle on a tree branch a few meters away from the old nest. There, they cluster around the queen, and a given set of bees (called *scout*) starts exploring the surrounding area. Each of the scout individuals finds a suitable location for the community to live: then, it returns to the rest of the population and performs the waggle dance to broadcast the information about the possible new nest, e.g., how suitable it is for the colony. Nest proposals coming from the scout bees may be different but, after some hours (sometimes days), an agreement is finally found. The whole swarm, organized in a well-defined pattern, finally takes off and flies towards the chosen destination, following the guidance of the scout/informed individuals (see Ref. [42] for more details).

Selected characteristics of such a bee collective migration are here reproduced by a microscopic-discrete approach. In particular, the component insects of a representative swarm are individually described by their position and velocity. Each bee is then set to behave following an individual strategy, i.e., the aim to reach a target destination, and social interactions. The latter involves repulsive/attractive stimuli (the desire to remain close to the rest of the population while maintaining a comfort space), as well as the ability of bees to synchronize their movement with a given set of surrounding individuals. Such insect behavioral rules have been of course previously proposed in the computational literature (cf. [17,18,26,28], see the conclusive section for more detailed comments): however, we here test and provide a comparison, in terms of numerical outcomes relative to selected swarming characteristics, of different combinations of alternative assumptions underlying flight alignment mechanisms and bee pairwise interactions. Further, theoretical results are used to derive proper parameter estimate.

Entering in more details, the rest of this paper is organized as follows. In Section 2, we clarify the assumptions on which our mathematical approach is based and present the model components. More specifically, we first explain the characteristic representation of model bees; then, we give the equations of motion and introduce the relative velocity components. Sections 3 deals with different assumptions underlying flight synchronization mechanisms. In particular, we focus either on an Euclidean metric-based or on a topological neighborhood metric-based alignment process within the swarm. In this respect, we discuss how these two mutually exclusive hypothesis impact on the repulsion/attraction velocity contributions (which in turn have to satisfy a stability condition to assure a realistic crystalline patterning of the particle system). Different series of numerical realizations then analyze the swarm behavior in different parameter regimes and show that our approach is able to capture selected experimentally-observed swarm phenomenology (e.g., flight synchronization and productive motion). After discussing in Section 4 the effect of the inclusion of random contributions on the particle system, we review in Section 5 the results obtained in this paper. Finally, we compare our approach with similar discrete models presented in the literature dealing with bee dynamics and propose some possible improvements and developments of the work.

2. Mathematical model

2.1. Bee representation and characteristics

A population of N bees is modeled in the two-dimensional space $\mathbb{R}^2$. We are indeed considering a planar section, parallel to the ground, of a typical swarm (see Fig. 1, left panel). Each bee $i = 1, \dots, N$ is intended as an autonomous particle and represented by a dimensionless material point with concentrated unitary mass. The generic i-th individual has the phase-space coordinates identified by $(\mathbf{x}_i(t), \mathbf{v}_i(t)) \in \mathbb{R}^{2\times 2}$, where the vectors $\mathbf{x}_i$, and $\mathbf{v}_i$ denote its actual position and velocity, respectively. We further define, for each insect i, a proper visual, as reproduced in Fig. 1, right panel:

$$\Omega_i^{\text{vis}}(t) = \left\{\mathbf{x} \in \mathbb{R}^2 : \left|\mathbf{x} - \mathbf{x}_i(t)\right| \le R^{\text{vis}}\right\}. \tag{1}$$

It is a round region centered in $\mathbf{x}_i(t)$ with radius R^{vis}, i.e., represents a vision depth which is hereafter set equal to 10 m according to the characteristic dimensions of swarm migration considered in this work.

2.2. Bee dynamics

The dynamics of a generic bee i can be described starting from a general second-order particle model:

$$m_i \frac{d^2\mathbf{x}_i}{dt^2}(t) + \lambda_i \frac{d\mathbf{x}_i}{dt}(t) = \mathbf{F}_i(t), \tag{2}$$

where m_i is the individual mass and λ_i a friction coefficient. $\mathbf{F}_i$ denotes instead the resultant of the forces that affect insect behavior. However, it is worth to notice that bees (such as most living entities, e.g., from cells and bacteria to big animal species and humans) are not passively prone to the Newtonian laws of inertia. They are in fact intelligent individuals able

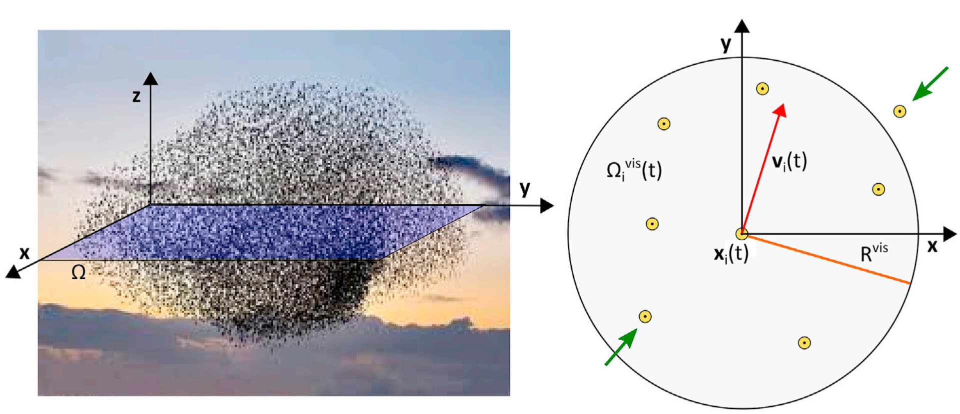

Fig. 1. Left panel: The virtual population of bees is modeled in the two-dimensional space $\mathbb{R}^2$, i.e., we are taking into account a planar section of a typical swarm. Right panel: Each bee i is represented as a material point and characterized by its actual position $\mathbf{x}_i(t)$ and velocity $\mathbf{v}_i(t)$. For each insect, we also define a visual region $\Omega_i^{\text{vis}}(t)$, i.e., a round area determined by the bee visual depth R^{vis}. The inclusion of a visual field implies that each bee is not able to see and therefore to interact with the entire set of their groupmates (see the individual indicated by the green arrow). For representative purposes, hereafter the virtual bees will be indicated by rigid disks centered at their actual position.