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Stock management in hospital pharmacy using chance-constrained model predictive control ☆

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ABSTRACT

One of the most important problems in the pharmacy department of a hospital is stock management. The clinical need for drugs must be satisfied with limited work labor while minimizing the use of economic resources. The complexity of the problem resides in the random nature of the drug demand and the multiple constraints that must be taken into account in every decision. In this article, chance-constrained model predictive control is proposed to deal with this problem. The flexibility of model predictive control allows taking into account explicitly the different objectives and constraints involved in the problem while the use of chance constraints provides a trade-off between conservativeness and efficiency. The solution proposed is assessed to study its implementation in two Spanish hospitals.

1. Introduction

Stock management is a common problem that is present in almost all the companies and organizations. The solution for this problem is given by a policy that determines how and when the orders should be placed. However, there are different difficulties associated to the problem. In the first place, there are uncertainties in the demand and delays in the deliveries, which make the problem not deterministic and require a degree of conservatism to avoid stockouts. It is needless to say that the lack of certain drugs in a hospital may endanger the life of the inpatients and, in the worst case, may have catastrophic consequences in the form of human losses. In order to avoid this situation, it is preferred to increase stock levels, but this is not always possible due to economical constraints. Actually, the pharmacy is a major source of expenses in hospitals. In [1], it is estimated that about 20–35% of the goods budget of a public hospital is spent by the pharmacy department. In a wider sense, the limitations imposed by the budget are also translated into the human resources in the pharmacy and the room available for storing drugs, which introduce additional constraints for the management. Hence, it may not be possible to place and receive orders too often due to the lack of pharmacists. Likewise, space constraints are important for example in drugs that must be stored in a fridge. Therefore, there is a need to develop advanced cost-efficient safe policies for stock management in hospitals capable of dealing with many different types of constraints.

In general, simple methods are used to solve inventory control problems. A usual policy is that of reorder point (s, S), that is, whenever the stock is below the level s, an order is placed to increase the stock up to the value S. Another option is to fix a size for the orders, Q, and submit an order once the stock is at level s. Other related policies about how to solve this problem are given in [2,3]. The major drawback with these techniques is that they are not able to take into account all the factors involved in the decision problem. In the literature, other alternatives are also proposed. For example, Bermejo et al. [4] presents an analytical model for the coordination of inventory and transportation in supply-chain systems. In [5], a supply chain network model consisting of manufacturers and retailers, where the demand is random, is developed.

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More strategies are presented in [6], where a competitive and cooperative selection of inventory policies in a supply chain with stochastic demand are studied. On the other hand, Rezapour and Farahani [7] develops a model to design a supply chain network with deterministic demand.

In this work, a model predictive control (MPC) strategy is proposed given the flexibility offered by its framework to handle multi-variable interactions, constraints on the problem variables, and optimization requirements in a systematic manner. Moreover, MPC has been successfully applied in the industry [8] and in similar problems. Some works based on the application of MPC are, for example, [9,10], where MPC is used to supply chain management in semiconductor manufacturing. Another example can be found in [11], where a distributed MPC algorithm with low communication burden is tested by using the MIT Beer Game (a supply chain benchmark). An extension of this scheme for systems with more than two controllers is also tested with supply chains in [12]. In [13], a robust MPC technique is used in a production-inventory system. Finally, in [14] the problem of managing inventories and supply chains is treated to reduce the number of tuning parameters with a technique based on a variation of MPC.

In the particular case of the stochastic control problems, i.e., those where the system being controlled is subject to uncertainties and/or unknown disturbances, the control policy can guarantee that the actual variables do not violate the constraints at the cost of an additional conservatism. That is, the actions implemented by the control policy are designed to deal with worst-case scenarios, which results in a waste of resources [15]. In this situation, it is acceptable to assume a low level of risk to save resources. To this end, the original constraints of the problem can be formulated in a probabilistic manner. The use of *chance constraints* was introduced in [16] and has been studied in a stochastic programming framework [17].

The implementation of MPC in combination with chance constraints is known as chance-constrained MPC (CC-MPC). The rationale of this approach is to replace hard constraints with probabilistic constraints and the nominal cost function with its expected value in the MPC formulation [18], leading to a stochastic optimization problem. CC-MPC offers advantages as robustness, flexibility, low computational requirements, and the possibility of including the level of reliability associated with the constraints [19,20]. Furthermore, since CC-MPC takes into account the expected performance of the closed loop with probabilistic constraints instead of directly trying to assure robust constraint satisfaction, it avoids the conservatism present in other robust MPC techniques, e.g.: [21,22].

There are other alternatives in the stochastic MPC literature that are also suitable for this type of problem. One option to deal with constraints on the inputs and states while optimizing some performance criterion, also in the presence of uncertainties or disturbances, is the scenario-based MPC method proposed in [23]. This method is based on the optimization of the control inputs over a finite horizon, subject to robust constraints under a finite number of random scenarios of the uncertainty and/or disturbances. A different but related approach is tree-based MPC [24], where the disturbances are grouped into a rooted tree that branches as the uncertainty grows. A tree of control actions is calculated to match the disturbance tree by the MPC controller. A simpler method is multiple MPC, which is given in [25], where control actions are calculated weighting the probability of occurrence of three possible scenarios. While all these approaches could be valid in for the problem considered in this article, they present some disadvantages with respect to CC-MPC. In the first place, scenario-based MPC requires a great amount of historical data to provide a low risk level. Tree-based MPC also requires a amount of historical data and solves a problem with a larger number of optimization variables, i.e., the computational burden of this method is greater. Finally, multiple MPC oversimplifies the computation of the control actions due to the low number of scenarios considered. In addition, all these methods have in common that the existence of a very severe scenario may result in an increase of conservativeness [26].

In this paper, which is an extension of the previous works [27,28], CC-MPC is used to solve the problem of inventory management in hospital pharmacies. In particular, the formulation of the problem is generalized with respect to the aforementioned works, where a Gaussian behavior of the demand is assumed. The mathematical development of the controller presented here can be applied even if the demand is only characterized statistically based on historical data. Likewise, the case of several hospitals that cooperate in order to relax their risk levels is another contribution of this work. It must be also remarked that this article has been carried out in the context of a project named *Pharmacontrol*, whose goal is to improve the inventory management in two Spanish hospitals.

The remainder of the paper is organized as follows. First, a description of the pharmacy inventory management optimization problem is shown in Section 2. Section 3 presents the MPC statement for this problem. In Section 4, some simulations are shown and the corresponding results are discussed. Finally, in Section 5 the conclusions are drawn.

2. Pharmacy inventory management

In this section, the mathematical background needed to build the optimization problem to be solved by the CC-MPC is presented.

2.1. System definition

In general, it will be assumed that there are N_i different drugs in the pharmacy inventory. The stock level of each one follows an evolution depending on the orders and on the demand. This evolution is represented by a discrete linear model, which for the particular case of drug i is

$$s_i(t+1) = s_i(t) + \sum_{j=1}^{np_i} o_i^j(t-\tau_i^j) - d_i(t), \tag{1}$$

where $s_i \in \mathbb{Z}$ is the stock of drug i, $o_i^j \in \mathbb{Z}$ is the number of ordered items to the j-th of the np_i providers of the drug i, τ_i^j is its corresponding transport delay, and $d_i(t)$ represents the aggregate demand of drug i.

The number of ordered items can be modeled as $o_i^j = \delta_i^j(t-\tau_i^j) o_i^j(t-\tau_i^j)$, where $\delta_i^j(t)$ is a Boolean variable, that is, $\delta_i^j(t) = 1$ if an order of drug i to provider j is placed during time t, otherwise $\delta_i^j(t) = 0$, and $o_i^j \in \mathbb{Z}$ represents the number of ordered items of drug i to provider j, only in those cases where $\delta_i^j(t) = 1$.

2.2. Single hospital optimization problem

The system can be represented according to Fig. 1. In this figure, the inputs represent the elements considered by the pharmacy managers to make the decisions about the order placement: the estimated demand, information about potential risks, and the constraints. The outputs are the optimal stock levels, minimum costs, and data about when and how many orders should be placed.

Every time an order is placed, the following costs are involved:

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