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Reconstruction of sparse-view X-ray computed tomography using adaptive iterative algorithms

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ABSTRACT

In this paper, we propose two reconstruction algorithms for sparse-view X-ray computed tomography (CT). Treating the reconstruction problems as data fidelity constrained total variation (TV) minimization, both algorithms adapt the alternate two-stage strategy: projection onto convex sets (POCS) for data fidelity and non-negativity constraints and steepest descent for TV minimization. The novelty of this work is to determine iterative parameters automatically from data, thus avoiding tedious manual parameter tuning. In TV minimization, the step sizes of steepest descent are adaptively adjusted according to the difference from POCS update in either the projection domain or the image domain, while the step size of algebraic reconstruction technique (ART) in POCS is determined based on the data noise level. In addition, projection errors are used to compare with the error bound to decide whether to perform ART so as to reduce computational costs. The performance of the proposed methods is studied and evaluated using both simulated and physical phantom data. Our methods with automatic parameter tuning achieve similar, if not better, reconstruction performance compared to a representative two-stage algorithm.

1. Introduction

X-ray computed tomography (CT) is an imaging technique widely used for medical diagnosis and treatments [1]. Due to potential risk of inducing secondary cancers, it is desirable to reduce radiation doses of X-ray CT imaging (e.g. fewer projection views, less angular coverage and lower incident X-ray intensity) [2]. For CT reconstruction with limited data, iterative reconstruction (IR) methods have demonstrated their capability of producing high quality images [3,4]. Thanks to advances of computer hardware and reconstruction algorithms, IR for CT has become a realistic approach in clinic practice [5].

One advantage of IR methods is that prior information can be incorporated with the imaging models, which can further improve reconstruction qualities [6–9]. In the past few years IR methods regularized by sparsity priors have been investigated [10–12]. One common approach is to minimize the total variation (TV), which is based on assumption of image sparsity in the gradient domain [13]. The nonlinear TV regularization can reduce noise while somewhat preserving edges of CT images.

Many iterative algorithms have been developed for regularized CT reconstruction [9,14–16]. A two-stage reconstruction approach was utilized in [17] where the data fidelity condition and prior constraints were enforced onto images separately. The same strategy was also adopted in TV regularized reconstruction [18–20]. In these iterative approaches, data fidelity constraints were enforced using the algebraic reconstruction technique (ART) and the TV objective was minimized using steepest descent or other optimization methods. To balance the two operations, a set of empirical parameters were introduced to adjust the contributions of the ART and TV optimization in [18]. However, one disadvantage of these iterative methods is that the parameters, such as step sizes, usually need to be manually tuned to achieve quick and convergent reconstruction and good image quality under different imaging conditions. Since this manual tuning process is tedious and time consuming, an automatic determination of reconstruction parameters for predictable results is highly desirable, which is the focus of this work.

For this purpose, a non-parametric control method was proposed to adjust the TV step size according to changes in the projection domain [21]. In this work, we propose to determine all iterative parameters automatically from data to avoid tedious manual parameter tuning. In TV minimization, the step sizes of steepest descent are adaptively adjusted according to the difference from POCS update in either the projection domain (projection controlled steepest descent, "PCSD") or the image domain (image controlled steepest descent, "ICSD"), while the step size of the ART in POCS is determined based on the data noise level.

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Furthermore, in two-stage approaches, the projections of the reconstructed image need to compare with the observed data to determine certain parameters [18]. This requires one full forward projection in addition to the forward and backward projections of the ART for each iteration. In this paper we utilize a novel mechanism to reduce the computational cost of two-stage CT reconstruction by skipping the ART operation for the data fidelity constraint when the projection error is not greater than a predefined error bound.

The rest of this paper is organized as follows. The objective function for X-ray CT reconstruction is provided in Section 2. The new adaptive iterative reconstruction algorithms are proposed in Section 3. In Section 4, performances of the proposed algorithms are evaluated using both simulated and real data. Discussion and conclusion are given in Section 5.

2. Constrained TV optimization for CT reconstruction

Given the system matrix $\mathbf{M}$, reconstruction of X-ray CT image $\mathbf{x}$, i.e. linear attenuation coefficient map, can be represented as optimization of the following regularized least-square model,

$$\mathbf{x}^* = \min_{\mathbf{x}} \left\{ \frac{1}{2} \|\mathbf{M}\mathbf{x} - \mathbf{p}\|^2 + \beta \|\mathbf{x}\|_{TV} \right\}, \tag{1}$$

where the first term in the model is a data fidelity term, $\boldsymbol{p}$ is the projection data whose element p_i is the logarithm of inversely normalized measured data on ith detector bin, and the second total variation (TV) term enforces edge-preserving smoothness penalty on reconstructed images [13]. TV is usually defined as,

$$\|\mathbf{x}\|_{TV} = \sum_{u,v} \left| (\nabla x)_{u,v} \right|, \tag{2}$$

where $\nabla \mathbf{x}$ is the gradient vector of $\mathbf{x}$. Its element $(\nabla x)_{u,v} = ((\nabla x)_u, (\nabla x)_v)$ with $(\nabla x)_u = x_{u+1,v} - x_{u,v}$ and $(\nabla x)_v = x_{u,v+1} - x_{u,v}$. $\left|(\nabla x)_{u,v}\right|$ is defined as,

$$\left| (\nabla x)_{u,v} \right| = \sqrt{((\nabla x)_u)^2 + ((\nabla x)_v)^2 + \delta}, \tag{3}$$

where δ is a small positive constant to smooth the non-differentiable TV norm as proposed in [10].

Eq. (1) is an unconstrained convex optimization problem and can be solved by well-established optimization techniques, such as conjugate gradient [19] and the first order primal dual methods [22]. However, the solution of Eq. (1) varies depending on the value of β that balances the contributions between TV and data fidelity terms. There is no straightforward way to determine its value for the optimal reconstruction other than trial-and-error tests. In this work, we focus on the parameter determination by treating sparse-view CT reconstruction as a constrained optimization as follows:

$$\mathbf{x}^* = \min_{\mathbf{x}} \|\mathbf{x}\|_{TV} \text{ subject to } \|\mathbf{M}\mathbf{x} - \mathbf{p}\|^2 \le \epsilon. \tag{4}$$

In this from, the objective is a TV minimization problem constrained by a data fidelity term in the projection domain. The parameter ϵ defines a certain amount of data error allowed between predicted and observed projection data due to noise and modeling errors, so called the "error bound" parameter. It is noted that there is an equivalence between the objective functions (1) and (4) when ϵ is not equal to zero. Nevertheless, the automatic determination of ϵ can be achieved using the projection data, thus avoiding the difficulty of the selection of β. For a well-calibrated CT system and well-compensated projection data, ϵ is dominated by the photon counting noise obeying Poisson distribution. Thus, an appropriate value for ϵ can be determined using the method proposed in [23]. The method approximates the noise variance of the measurement y_i for the line integral along the ith ray, the received X-ray intensities on the ith pixel of detectors,

$$\epsilon_i = \frac{1}{y_i}. \tag{5}$$

Given incident photon intensity y_0, $p_i = \ln(y_0/y_i)$ based on the Beer's law. ϵ is then estimated as the summation over all projections

$$\epsilon = \sum_i \epsilon_i. \tag{6}$$

In practice, a non-negative constraint on image pixels (linear attenuation coefficients) is also introduced and leads to the following constrained optimization problem,

$$\mathbf{x}^* = \arg\min \|\mathbf{x}\|_{TV} \text{ subject to } \|\mathbf{M}\mathbf{x} - \mathbf{p}\|^2 \le \epsilon \text{ and } \mathbf{x} \ge 0, \tag{7}$$

3. The reconstruction algorithms

The constrained optimization problem can be solved using a two-stage framework: (1) the enforcement of data fidelity and non-negativity constraints, and (2) the minimization of TV [10]. Two operations are iterated in alternation to achieve the optimal solution. Projection onto convex sets (POCS) is used at the first stage by treating constraint sets as convex sets and selecting a feasible image through projections onto these sets. Iterative methods such as steepest descent can be applied for TV optimization. Such TV-POCS approaches usually include two loops of iteration: a main loop of POCS and a sub-loop of TV optimization. The parameters that control the step sizes of POCS and TV minimization are important for a well-behaved iterative reconstruction. In this work, we proposed automatic approaches to determine these parameters to avoid tedious trial-and-error parameter tuning.

3.1. Projection onto convex sets (POCS) of constraints

In the POCS stage, the feasible solution of data fidelity term can be obtained using the algebraic reconstruction technique (ART). For the ith ray of X-ray CT, the original ART from an initial point $\mathbf{x}_s$ is the pure projection $\mathrm{P}\mathbf{x}_s$ onto the hyperplane defined by $\mathbf{m}_i\mathbf{x} - p_i = 0$,

$$\mathbf{x} = \mathrm{P}\mathbf{x}_s = \mathbf{x}_s + \frac{1}{\|\mathbf{m}_i\|^2}(p_i - \mathbf{m}_i\mathbf{x}_s)\mathbf{m}_i^T, \tag{8}$$

where $\mathbf{m}_i$ is the ith row of the system matrix $\mathbf{M}$. The original projection form of ART is suitable for noiseless data where the set of linear equations is consistent. To accommodate noisy data fidelity constraint, a relaxed projection form of ART is introduced,

$$\mathbf{x} = (1-\lambda_i)\mathbf{x}_s + \lambda_i \mathrm{P}\mathbf{x}_s = \mathbf{x}_s + \frac{\lambda_i}{\|\mathbf{m}_i\|^2}(p_i - \mathbf{m}_i\mathbf{x}_s)\mathbf{m}_i^T, \tag{9}$$

where λ_i is the relaxation parameter [24].

In this study we proposed to determine the relaxation parameter according to the noise level of X-ray photon detection, whose variance can be approximated as the inverse of the detected counts shown in Eq. (5) [25–27]. Specifically, the relaxation parameter can be defined as the normalized X-ray intensities,

$$\lambda_i = \sqrt{\frac{y_i}{y_0}} = \sqrt{\exp(-p_i)}. \tag{10}$$

As can be seen, λ_i is confined between [0, 1]. Geometrically, the relaxed point $\mathbf{x}$ can be interpreted as an interior point on the line segment from the initial point $\mathbf{x}_s$ to the projection $\mathrm{P}\mathbf{x}_s$, and λ_i is a weight of projection point. When the projection data has the lowest noise level, i.e. $y_i = y_0$ or no attenuation, $\lambda_i = 1$ will lead to the pure projection $\mathrm{P}\mathbf{x}_s$. On the other hand, to account for the variability of the observation p_i, the relaxation parameter λ_i will relax the projection to $(1-\lambda_i)\mathbf{x}_s + \lambda_i \mathrm{P}\mathbf{x}_s$. The higher the noise level,