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Research paper

Fast matrix inversion and determinant computation for Polarimetric Synthetic Aperture Radar

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ABSTRACT

This paper introduces a fast algorithm for simultaneous inversion and determinant computation of small sized matrices in the context of fully Polarimetric Synthetic Aperture Radar (PolSAR) image processing and analysis. The proposed fast algorithm is based on the computation of the adjoint matrix and the symmetry of the input matrix. The algorithm is implemented in a general purpose graphical processing unit (GPGPU) and compared to the usual approach based on Cholesky factorization. The assessment with simulated observations and data from an actual PolSAR sensor show a speedup factor of about two when compared to the usual Cholesky factorization. Moreover, the expressions provided here can be implemented in any platform.

1. Introduction

Microwave remote sensing is basilar as it provides complementary information to that provided by classical sensors which perceive the optical spectrum. Longer wavelengths in the 1 cm to the 1 m can penetrate clouds and other adverse atmospheric conditions, as well as canopies and soils. There are passive and active microwave sensors; the latter carry their own source of illumination, and can be either imaging or non-imaging. Radar devices belong to the former. They transmit a radio signal and detect the returned echo (backscatter), with which an image is formed. Several signal processing techniques are used to enhance the spatial resolution of such imagery. In particular, the use of synthetic antennas leads to synthetic aperture radar (SAR) imaging; as well as to polarimetric SAR (PolSAR) imaging which employs polarized signals.

The statistical properties of PolSAR were studied in the context of optical polarimetry (Goodman, 1985). The simplest case occurs when the backscatter is constant. In such a case, the targets are characterized by a scattering (or Sinclair) matrix $\mathbf{S}$, which describes dependence of its scattering properties on the polarization. The scattering matrix is defined on the horizontal (H) and vertical (V) basis as

$$\mathbf{S}' = \begin{bmatrix} S_{\mathrm{HH}} & S_{\mathrm{HV}} \\ S_{\mathrm{VH}} & S_{\mathrm{VV}} \end{bmatrix}, \tag{1}$$

where each element is a complex value, representing the amplitude and the phase of the scattered signal. Reciprocity $S_{\mathrm{HV}} = S_{\mathrm{VH}}$ holds for most natural targets, so one may use the scattering vector $\mathbf{S} = [S_{\mathrm{HH}} \;\; S_{\mathrm{HV}} \;\; S_{\mathrm{VV}}]^{\top}$ without loss of information, where $\top$ represents transposition (Anfinsen et al., 2011).

More often than not, these single-look complex data are processed in order to improve the signal-to-noise ratio. Multilook fully polarimetric data are formed as

$$Z^{(N)} = \frac{1}{N} \sum_{\ell=1}^{N} \mathbf{S}(\ell)^{\mathrm{H}} \mathbf{S}(\ell), \tag{2}$$

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where ^{H} denotes the conjugate transposition and ℓ indexes theoretically independent looks of the same scene. This 3×3 complex matrix is positive Hermitian with real entries in the diagonal.

As noted by Torres et al. (2014) and the references therein, many techniques for PolSAR image processing and analysis rely on the statistical properties of $Z^{(N)}$. The density of several models (Wishart (Goodman, 1963), K (Yueh et al., 1990), G^0 (Frery et al., 1997), G (Freitas et al., 2005), Kummer-U (Doulgeris, 2015) to name a few) depends solely on a few parameters: the covariance matrix, the number of looks L and, sometimes, texture descriptors. The covariance matrix is the expected value of $Z^{(N)}$, namely $\mathbf{Z} = \mathrm{E}\{Z^{(N)}\}$, and it enters the expressions only through its determinant and its inverse.

Moreover, divergences among these models (Kullback-Leibler, Hellinger, Rényi, Bhattacharya, Triangular, Harmonic (Nascimento et al., 2010)), test statistics (likelihood ratio) (Conradsen et al., 2003), and classification rules (Skriver) only require the computation of $\det(\mathbf{Z})$, $\mathbf{Z}^{-1}$, and $\det(\mathbf{Z}^{-1})$. A typical Uninhabited Aerial Vehicle Synthetic Aperture Radar (UAVSAR) image may have $10^4 \times 4{\cdot}10^4$ pixels, where each pixel is represented by a 3×3 Hermitian matrix as in (2). As illustrative example, if one relies on Nonlocal Means Filter approach based on stochastic distances (Torres et al., 2014), at each sear window, which are typically large, e.g. 23×23 pixels, one needs to compute 23^2 inverse and 23^2 determinant operations for each pixel. This scales to a long computing time as it results in a total of $23^2{\cdot}4{\cdot}10^8 \approx 2.1{\cdot}10^{11}$ inverse matrix and determinant calculations *per image*. Such amount of data is likely to increase with the incoming availability of sensors with finer resolution. Thus one reaches the conclusion that accurate and fast procedures are of paramount importance for dealing with PolSAR data.

The Cholesky factorization is the most popular numerical analysis method for the direct solution of linear algebra tasks involving positive definite dense matrices (Björck, 2014; Golub and Van Loan, 1996; Shores, 2007). It is also the algorithm of choice for matrix inversion and determinant calculation in the context of image classification of PolSAR images (Torres et al., 2014). In this paper, we propose a fast algorithm for the computation of matrix inverse and determinant of 3×3 Hermitian matrices in the context of PolSAR image classification that outperforms the Cholesky factorization. The introduced algorithm is proven to reduce the overall arithmetic complexity associated with the matrix inversion and determinant calculation when compared with the Cholesky factorization approach. Such lower arithmetic cost results in a reduction of the computation time to about a half of the Cholesky based method. The proposed algorithm and the matrix inversion and determinant calculation based on Cholesky factorization are implemented in a general purpose graphical processing unit (GPGPU) using C/C++ and open computing language (OpenCL), which are tools that have been used for accelerating a several of algorithms in geoscience and remote sensing (Lukač and Žalik, 2013; Steinbach and Hemmerling, 2012; Li et al., 2014, 2015). The proposed algorithm and the method based on Cholesky factorization are tested using both simulated and measured PolSAR data (The Polarimetric SAR Data Processing and Educational Tool, 2017).

The paper unfolds as follows. Section 2 introduces notation and preliminary considerations. The Cholesky factorization is reviewed and described as a means for matrix inversion and determinant computation. In Section 3, the proposed method and its fast algorithm is introduced. Section 3 brings the arithmetic complexity assessment and a discussion on the numerical error analysis of the proposed method compared with the Cholesky factorization approach. Section 4 has implementation considerations including software and hardware comments. Experiment results shows the effectiveness of the proposed method. Final comments and suggested directions for future works are in Section 5.

2. Mathematical review

2.1. Preliminaries and notation

The type of matrix that occurs in the PolSAR problem is the 3×3 correlation matrix with complex entries. Being a correlation matrix, it inherits the Hermitian property (Golub and Van Loan, 1996). Therefore, it can be represented according to:

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ \bar{b} & d & e \\ \bar{c} & \bar{e} & f \end{bmatrix},$$

where a, d, and f are real quantities; b, c, and e are complex numbers; and the overbar *bar* denotes the complex conjugation. Because $\mathbf{A}$ is Hermitian, in actual implementations, only the upper or lower triangular part of the matrix is stored for computation.

The (i, j) cofactor of the matrix $\mathbf{A}$ is the determinant of the submatrix formed with the elimination of the ith row and jth column times $(-1)^{i+j}$ (Golub and Van Loan, 1996). The adjoint matrix is computed according to the following (Golub and Van Loan, 1996):

$$\widetilde{\mathbf{A}} = \begin{bmatrix} a_c & b_c & c_c \\ \bar{b}_c & d_c & e_c \\ \bar{c}_c & \bar{e}_c & f_c \end{bmatrix},$$

where the element in position (i, j) represents the cofactor of the element (j, i) in the original matrix $\mathbf{A}$ and are given by

$$\begin{aligned} a_c &= d{\cdot}f - |e|^2, \\ b_c &= c{\cdot}\bar{e} - b{\cdot}f, \\ c_c &= b{\cdot}e - c{\cdot}d, \\ d_c &= a{\cdot}f - |c|2, \\ e_c &= c{\cdot}\bar{b} - a{\cdot}e, \\ f_c &= a{\cdot}d - |b|^2. \end{aligned} \tag{3}$$

The adjoint matrix $\widetilde{\mathbf{A}}$ is also Hermitian; thus a_c, d_c, and f_c are real numbers.

2.2. Cholesky factorization

Cholesky factorization is applied in many numerical problems (Aquilante et al., 2008; Wilson, 1990; Kershaw, 1978). Let $\mathbf{A}$ be the input matrix we are interested into inverting and computing the determinant. Cholesky factorization decomposes an input matrix into the product $\mathbf{L}{\cdot}\mathbf{L}^{\mathrm{H}}$, where $\mathbf{L}$ is a lower triangular matrix and ^{H} represents the transpose conjugate operation. Let $l_{i,j}$ be the (i, j) entry of $\mathbf{L}$. The Cholesky factorization is based on the following relations between the elements of $\mathbf{A}$ and $\mathbf{L}$:

$$\begin{aligned} |l_{0,0}|^2 &= a, \\ l_{1,0}{\cdot}l_{0,0} &= \bar{b}, \\ l_{2,0}{\cdot}l_{0,0} &= \bar{c}, \\ |l_{1,1}|^2 + |l_{1,0}|^2 &= d, \\ l_{2,1}{\cdot}l_{1,1} + l_{1,0}{\cdot}\bar{l}_{2,0} &= e, \\ |l_{2,2}|^2 + |l_{2,0}|^2 + |l_{2,1}|^2 &= f, \end{aligned}$$

where $|\cdot|$ returns the magnitude of its complex argument (Graham et al., 1989).

Once matrix $\mathbf{L}$ is derived, its inverse can be directly obtained from the following expressions:

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