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Research paper

A structural rank reduction operator for removing artifacts in least-squares reverse time migration

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ABSTRACT

Least-square reverse time migration (LSRTM) has been widely accepted because of its exceptional performance in mitigating migration artifacts and preserving the reflection amplitude. Due to the ill-posedness of the inverse problem, regularization methods or constraints must be applied to the reflectivity model. In this paper, we propose a novel iterative LSRTM framework that is regularized by a lowrank constraint. The lowrank constraint is applied along the geological structure of the subsurface reflectivity image and thus can also be called structural lowrank constraint. The lowrank constraint is applied by iteratively applying a rank reduction operator that is based on the lowrank approximation theory. The rank reduction operator applied along the structure direction can effectively remove those artifacts caused from sparse shot/receiver sampling or other circumstances. Compared with the traditionally used smoothness based constraint, the lowrank constraint is more capable of removing noise while preserving edge details. Since the constraint is applied in post-stack seismic image, the extra computational cost caused by the singular value decomposition (SVD) of the rank reduction operator is negligible compared with the computational cost of the migration operator. Numerical examples with different levels of structural complexity are used to demonstrate the effectiveness and validity of the proposed algorithm.

1. Introduction

Seismic migration (imaging) is a process to map the multi-dimensional seismic data onto a 2D/3D image for subsurface characterization (McMechan, 1983; Levin, 1984; Chang and McMechan, 1989, 1986; 1987, 1994; Hubral et al., 1996; Hokstad et al., 1998; Hokstad and Sollie, 1998; Hokstad, 2000; Sun et al., 2006). From the mathematical point of view, seismic migration can be treated as a linear inverse problem (Chen et al., 2017c). Traditional migration applies the adjoint operator, instead of an inverse operator, to the observed data (Wu and Bai, 2018). Thus, the subsurface reflectivity image resulted from the traditional migration method will inevitably contain artifacts or are not of true amplitude (Zhang et al., 2015). These artifacts are caused by a complicated mechanism that is related with a variety of reasons, e.g., sparse shot/receiver sampling, narrow shot-receiver apertures, and limited signal bandwidth. A better way to compute the subsurface image is by inverting the forward operator (or demigration operator) in the linear inverse problem, which is called least-squares migration (LSM) (Ronen and Liner, 2000). Due to the extremely large size of the forward operator and the ill-poseness of the inversion problem, an iterative solver together with some regularization constraints must be utilized (Xue et al., 2016c; Dutta, 2016). Due to different migration operators, the LSM can be grouped into several types. The least-squares Kirchhoff migration takes advantage of the ray-tracing strategy for the migration operator and is computationally efficient (Nemeth et al., 1999; Duquet et al., 2000). The least-squares one-wave wave equation migration uses the one-wave wave equation operator as the migration operator (Kuehl and Sacchi, 2003; Wang et al., 2005; Clapp et al., 2005). Recently, the two-way reverse time migration operator is utilized in the least-squares inversion framework due to the fast development of modern computing architecture (Chen et al., 2017b).

The least-squares reverse time migration (LSRTM) method can not

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only help reduce migration artifacts, improve the amplitude fidelity of reflectivity images, but also can help better image the steeply dipping reflectors than other LSM methods. The LSRTM has been applied to migrate the incomplete data and simultaneous-source seismic data (Xue et al., 2016c). A cross-correlation based objective function was proposed in Zhang et al. (2015), instead of the normally used L_2 norm amplitude misfit objective function, to relax the amplitude constraints and thus can be robustly applied to real data with a stable performance. A source-independent LSRTM based on a convolution-based objective function was proposed in Zhang et al. (2016c) to relieve the dependence of traditional LSRTM on an accurate estimation of source wavelet. Feng and Schuster (2017) extended the acoustic LSRTM to elastic LSRTM by substituting the acoustic wave equation involved in the acoustic LSRTM with the linearized elastic wave equation. Feng and Schuster (2017) showed that the elastic LSRTM images have better resolution and amplitude balancing, fewer artifacts, and less crosstalk compared with the elastic RTM images. The images are also better focused and have better reflector continuity for steeply dipping events compared to the acoustic LSRTM images. However, Feng and Schuster (2017) also pointed out that the elastic LSRTM set a higher demand on the migration velocity analysis since more artifacts will exist when an inappropriate subsurface macro velocity model is obtained.

The LSRTM cannot obtain successful performance without proper regularization on the inverse problem (Xue et al., 2016c). Because of the continuous property in the reflection angle dimension, angle-domain common-image gathers (Sava and Fomel, 2003, 2006) can be straightforwardly utilized for applying some constraints that are based on the spatial continuity (Xue et al., 2016c). A smoothing regularization can be applied to suppress migration artifacts and have be demonstrated to be effective in many applications (Kuehl and Sacchi, 2003; Prucha and Biondi, 2002). The inconsistency between neighbor traces in the angle gathers can be damped in order to compensate for the subsurface illumination (Salomons et al., 2014). Other type of filtering methods are also possible to remove the artifacts during LSRTM, such as the mode decomposition based methods (Chen and Ma, 2014; Liu et al., 2016a, 2017; Chen, 2016, 2018b; Wu et al., 2018), sparse transform based filtering methods (Liu et al., 2016b; c), statistics-based methods (Yang et al., 2015; Bai and Wu, 2017; Huang et al., 2018b; Xie et al., 2018), inversion-based filtering methods (Chen and Fomel, 2015; Jiao et al., 2015; Chen and Jin, 2015), mathematical morphology based methods (Li et al., 2016a; b; Huang et al., 2017a; c; 2018a), etc. The angle domain constraint based on dip filtering was used in Stanton and Sacchi (2015) for LSM of elastic data. A robust hybrid norm objective function was used in Zhang et al. (2016c) for constraining the migration inversion in the situations of strong random Gaussian noise and spiky noise. Structural smoothness based regularization algorithms are used in Xue et al. (2016c) with the preconditioned LSRTM framework or shaping regularized based LSRTM framework. During iterative inversion, in order to remove artifacts, a smoothing operator can be applied along the dip angle direction (Xue et al., 2016c). However, it is challenging or even impossible to choose an appropriate smoothing window length in order to obtain the best compromise between noise removal and amplitude preservation. A L_1 regularization term is added to the objective function in Wu et al. (2016a) for constraining the reflectivity model to be sparse in some transformed domains, e.g., Fourier transform domain (Pratt et al., 1998; Li et al., 2016c; Zhong et al., 2016; Zhou et al., 2017b), curvelet domain (Zu et al., 2016; Liu et al., 2018b; Zhao et al., 2018), seislet domain (Fomel and Liu, 2010; Gan et al., 2015, 2016b; 2016a; Chen and Fomel, 2018), adaptively learned sparse dictionary domain (Rubinstein et al., 2008; Chen, 2017; Chen et al., 2016b; Siahsar et al., 2017; Zhou et al., 2017a), and Radon domain (Xue et al., 2016b; Chen, 2018a). Lin and Huang (2015) proposed a modified total-variation (TV) regularization based LSRTM method to enhance image quality and reduce artifacts. This TV based method makes use of Tikhonov regularization and classic TV regularization. The modified TV regularization objective function is solved with two splited subproblems based on the preconditioned conjugate-gradient and split Bregman iterative methods.

One of the most commonly used regularization methods for LSRTM is based on smoothing. Smoothing, however, will inevitably cause amplitude loss for edges. In this paper, a new regularization algorithm is proposed based on the lowrank assumption (Xue et al., 2016a; Chen et al., 2016c; Zhou et al., 2018) of reflectivity images along the structure direction. The new regularization method can solve the edge damage problem caused in the smoothing based regularization to some extent. A rank reduction operator is applied along the structural direction of the subsurface reflectivity image. The structural information is obtained from an initial guess of the underground geological structures resulted from a simple RTM based migration approach. We use plane-wave destruction algorithm (Claerbout, 1985) to calculate the local slope that is required by the structural rank reduction operator. The lowrank constraint is superior than the smoothness constraint in that it can help preverse amplitude variations of reflection events and better preserve the discontinuities. Different synthetic examples with increasing complexities are used to illustrate the effectiveness of the proposed algorithm.

The paper is organized as follows: we first introduce the general iterative algorithm with a constraining operator to solve the LSRTM related inverse problem, secondly we introduce in detail the rank reduction operator which is the key in the lowrank LSRTM framework and provide the pseudo-codes for easier implementation, thirdly we use multiple examples with different levels of complexity to demonstrate the superior performance of the proposed algorithm than the traditional algorithm, and finally we draw some conclusions in the end of the paper.

2. Theory

2.1. Least-squares reverse time migration (LSRTM)

In a matrix-vector form, the forward modeling of reflection seismic data using Born approximation mentioned above can be simply expressed as

$$\mathbf{L}\mathbf{r} = \mathbf{d}, \tag{1}$$

where $\mathbf{d}$ is the traditionally recorded data, $\mathbf{r}$ denotes the reflectivity model, and $\mathbf{L}$ denotes the Born modeling operator (Tarantola, 1984). Conventional migration methods seeks to approximate the inverse of operator $\mathbf{L}$ by its adjoint, and the resulted model $\widehat{\mathbf{r}}$ is the traditionally migrated data:

$$\widehat{\mathbf{r}} = \mathbf{L}^T\mathbf{d}, \tag{2}$$

where $\mathbf{L}^T$ denotes the adjoint of the forward operator. LSRTM seeks to invert the $\mathbf{L}$ for obtaining $\mathbf{r}$ while using the two-way wave equation to propagate seismic waves. Direct inversion of equation (2) is not possible because of the extremely large forward matrix and the serious ill-posedness of the inverse problem. Instead, we can use iterative method to solve equation (2). The iterative updating formula can be

$$\mathbf{r}_{n+1} = \mathbf{r}_n + \alpha_n\mathbf{s}_n, \tag{3}$$

where $\mathbf{r}_n$ is the updated model after n iterations, α_n and $\mathbf{s}_n$ is the step size and updating direction, respectively. α_n can be obtained by performing a line minimization along a given direction $\mathbf{s}_n$. The direction $\mathbf{s}_n$ can be chosen using the following relation between $\mathbf{s}_n$ and $\mathbf{s}_{n-1}$:

$$\mathbf{s}_n = \mathbf{f}_n + \beta_n\mathbf{s}_{n-1}, \tag{4}$$

where $\mathbf{f}_n$ is a given random direction and β_n is chosen such that $\mathbf{s}_n$ and $\mathbf{s}_{n-1}$ are conjugate. If $\mathbf{f}_n$ is chosen as the gradient direction such that $\mathbf{f}_n = \mathbf{L}^T(\mathbf{d} - \mathbf{L}\mathbf{r}_n)$, the iterative approach of equations (3) and (4) is the so-called conjugate gradient (CG) algorithm (Hestenes and Stiefel, 1952). However, due to strong migrated artifacts due to sparse shot/receiver samplings and narrow migration apertures, each updated $\mathbf{r}_n$ may deviate