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Research paper

A case study of forward calculations of the gravity anomaly by spectral method for a three-dimensional parameterised fault model

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ABSTRACT

Spectral methods provide many advantages for calculating gravity anomalies. In this paper, we derive a kernel function for a three-dimensional (3D) fault model in the wave number domain, and present the full Fortran source code developed for the forward computation of the gravity anomalies and related derivatives obtained from the model. The numerical error and computing speed obtained using the proposed spectral method are compared with those obtained using a 3D rectangular prism model solved in the space domain. The error obtained using the spectral method is shown to be dependent on the sequence length employed in the fast Fourier transform. The spectral method is applied to some examples of 3D fault models, and is demonstrated to be a straightforward and alternative computational approach to enhance computational speed and simplify the procedures for solving many gravitational potential forward problems involving complicated geological models. The proposed method can generate a great number of feasible geophysical interpretations based on a 3D model with only a few variables, and can thereby improve the efficiency of inversion.

1. Introduction

The gravimetry methods employed in physical geodesy, which is a very important branch of geophysics, seek to evaluate and interpret gravity anomalies that can provide considerable information regarding the hidden structures below the earth surface, such as the depth of geological bodies and the geometry and density variations in the structure of the subsurface (Hinze et al., 2013). The interpretation and inversion of gravity anomalies are generally based on the understanding obtained from theoretical mass distribution models, which are developed according to the forward problem of computing the gravitational field obtained from a particular mass distribution. Analytical expressions for theoretical gravity anomaly distributions (or maps) are easily derived from idealized bodies, and can help to simplify practical geological and geophysical problems. For example, the theoretical gravity anomalies obtained from idealized dike, cylinder, fault, and geological contact models are essential for understanding gravimetry measurement results (Sengupta and Das, 1977; Soto et al., 1983; Sharma and Bose, 1977; Chacko and Bhattachyya, 1980). Usually, the gravity anomaly map obtained from a two-dimensional (2D) or three-dimensional (3D) model can be calculated in the space domain by integration if the boundaries of the model are known. Accordingly, many useful forward analytical equations have been presented in the space domain (Talwain and Ewing, 1960; Nagy, 1966). Nevertheless, complex and irregular idealized models can greatly complicate the derivation of analytical expressions in the space domain. However, the use of the Fourier transform method presents spectral techniques that can provide for the more simple and rapid computation of gravity anomalies in the wave number domain than in the space domain (Bhattacharyya and Navolio, 1976). Related applications in the wave number domain have been numerous and well developed (Bhattacharyya, 1967; Cassano and Rocca, 1975).

Compared with the interpretation of idealized bodies in the space domain, the use of spectral methods provides many interpretational advantages because forward gravitational field problems become very straightforward, and are rapidly computed when transformed from the space domain to the wave number domain. For example, convolution in the space domain becomes multiplication in the wave number domain. Moreover, the gradient operation in the space domain is only related to the angular frequency in the wave number domain. Spectral methods have been widely employed for the analysis of gravitational fields in numerous studies (Sharma et al., 1970; Regan and Hinze, 1976, 1977, 1978; Bhattacharyya and Leu, 1977), and have proved very useful, particularly for improving the computing speed and gravity anomaly transformations. Parker (1973) presented a well-known rapid spectral

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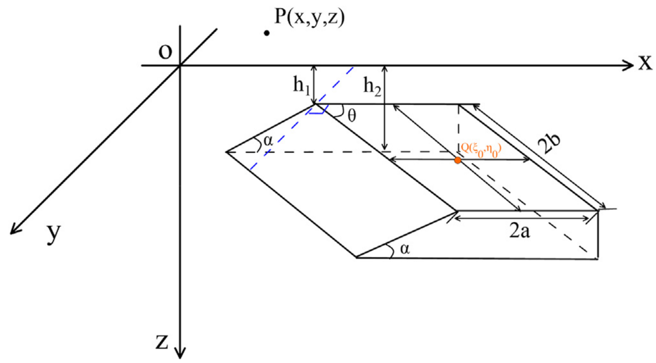

Fig. 1. Schematic of the idealized 3D fault model. The model is described in terms of 3D Cartesian coordinates, where the positive z-direction is downward. The parameters α and θ represent the dip and strike angles, respectively. The parameters h_1 and h_2 represent the depths of the top and bottom surfaces, respectively. The values $2a$ and $2b$ express the width and length of the model, respectively. The point P(x,y,z) is the observation point of the gravity anomaly. The point Q(ξ_0, η_0, ζ_0) is the source location of the model center.

Table 1
Comparison of computation times between the space and wave number domian.

Case	FFT Sequence lengths	Space domain computing time (s)*	Frequency domain computing time*	
			Spectrum time (s)	IFFT time (s)
1	256 × 256	0.125	0.016	0.078
2	512 × 512	0.406	0.141	0.641
3	1024 × 1024	1.828	0.906	2.703

*The computing time tested by the laptop with Intel Core i5-3360M CPU 2.8 GHz.

method for the forward computation of gravity anomalies obtained from an uneven layer of material, and this approach is typically employed to estimate the thickness of crust and subsurface undulation. This is one of the most important methods in gravitational field studies because the forward problem for an uneven subsurface structure can be calculated by means of a Taylor series in the wave number domain rather than using the integration of a large number of rectangular prisms. The iterative inversion approach was also developed on the basis of Parker's formula (Oldenburg, 1974), and numerous studies have implemented and discussed this algorithm (Gomez-Ortiz and Agarwal, 2005; Shin et al., 2006). However, except for the forward problem applied to complex idealized geometry models, few studies have proposed direct forward approaches in the wave number domain. Moreover, analytical forward equations for a non-simplified 3D fault model have also been rarely studied.

In the present study, we derive an analytical equation and demonstrate a forward computation approach for a non-simplified 3D fault model in the wave number domain. Our approach is applied to several examples of the 3D fault model for the purpose of error analysis and to illustrate its use in solving realistic problems. The proposed forward approach was programmed using the Fortran language, which has been extensively employed in geophysical research (Shin et al., 2006; Fernández et al., 2008). The program code developed here is expected to be very useful for many researchers, particularly for those researchers who prefer to code using the Fortran language. Finally, we also discuss further applications using the presented 3D fault model.

2. Forward computation of gravity anomalies in the wave number domain

A gravity anomaly is the vertical derivative of the gravitational potential U, which, in the space domain, can be expressed as

$$U = G\iiint \frac{\Delta\rho}{r}\mathrm{d}V, \tag{1}$$

where G is Newton's gravitational constant, $\Delta\rho$ is the density contrast, and r is the distance between the field source and the point of observation. If we take a point of observation P at coordinates (x, y, z) and an elementary mass Q having coordinates (ξ, η, ζ), the elementary volume $\mathrm{d}V = \mathrm{d}\xi\mathrm{d}\eta\mathrm{d}\zeta$ and $r = \sqrt{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2}$. The 2D Fourier transform $\mathscr{F}[\cdot]$ of Eq. (1) can be written as

$$\mathscr{F}[U] = \iint U e^{-i2\pi(ux+vy)}\mathrm{d}x\mathrm{d}y, \tag{2}$$

where $i = \sqrt{-1}$, and u and v are the angular frequencies in the x and y directions, respectively. In order to understand how potential field anomalies can be calculated in the wave number domain, Parker (1973) has shown how a series of Fourier transforms can be used to calculate the magnetic or gravitational anomaly. In this paper, we replace $\mathscr{F}[U]$ with $U(u,v)$ to express the 2D spectrum of $U(x,y)$. If P is confined to the plane $z = z_0$, the gravitational potential can be written in the wave number domain as

$$U(u, v) = \frac{2\pi G\Delta\rho}{\omega}e^{\omega z_0}\iiint e^{-\omega\zeta}e^{-2\pi i(u\xi+v\eta)}\mathrm{d}\xi\mathrm{d}\eta\mathrm{d}\zeta, \tag{3}$$

where ω is the angular frequency, and $\omega = \sqrt{u^2 + v^2}$. The integral of Eq. (3) can be conducted analytically as the 2D spectrum of the gravitational potential for a known mass shape. A well-known straightforward relationship exists between the gravitational potential and its derivative in the wave number domain:

$$\mathscr{F}[U_z] = \omega\cdot\mathscr{F}[U]; \quad \mathscr{F}[U_x] = iu\cdot\mathscr{F}[U]; \quad \mathscr{F}\left[U_y\right] = iv\cdot\mathscr{F}[U]. \tag{4}$$

Moreover, the gradient tensors of U can be easily derived, where the 2D spectrum expressions of the five independent gravity gradient tensors can be written as follows:

$$\begin{aligned}&\mathscr{F}[U_{xz}] = iu\omega\cdot\mathscr{F}[U]; \mathscr{F}\left[U_{yz}\right] = iv\omega\cdot\mathscr{F}[U]; \quad \mathscr{F}\left[U_{xy}\right] = -uv\cdot\mathscr{F}[U];\\ &\mathscr{F}[U_{xx}] = -u^2\cdot\mathscr{F}[U]\ ; \ \mathscr{F}\left[U_{yy}\right] = -v^2\cdot\mathscr{F}[U].\end{aligned} \tag{5}$$

The corresponding gravity anomaly in the space domain can be easily obtained from the gravitational potential spectrum or its derivative by means of the inverse Fourier transform.

In this study, we define a type of non-simplified 3D fault model, which is illustrated in Fig. 1, and an analytical expression for this model is derived in the wave number domain from Eq. (3). First, we define the xy plane as 0, i.e., $z_0 = 0$, and introduce the kernel function $E(u,v)$ as follows:

$$\mathscr{F}[U(x, y)] = U(u, v) = \frac{2\pi G\Delta\rho}{\omega}E(u, v). \tag{6}$$

We can express the limits of integration for this 3D model as

$$E(u, v) = \int_{h_1}^{h_2}\int_{\eta_0-bsin\theta}^{\eta_0+bsin\theta}\int_{\xi_0-a-(\zeta-h_1)cot\alpha-(\eta-\eta_0)cot\theta}^{\xi_0+a-(\eta-\eta_0)cot\theta}e^{-\omega\zeta}e^{-i(\xi u+\eta v)}\mathrm{d}\xi\mathrm{d}\eta\mathrm{d}\zeta, \tag{7}$$

where the parameters h_1, h_2, a, b, θ, and α, and the reference point Q(ξ_0, η_0, ζ_0) of the model are illustrated in Fig. 1. Computing the integral of Eq. (7) for ξ, η, and ζ, respectively, where related details are presented in the Appendix, yields the following.

$$E(u, v) = E_0(u, v)e^{-i(\xi_0 u+\eta_0 v)}$$

$$E_0(u, v) = \frac{-2isin[bsin\theta(v - ucot\theta)]}{u(v - ucot\theta)}\left\{\frac{(e^{-\omega h_2} - e^{-\omega h_1})e^{-iua}}{\omega} + \frac{e^{iua}\left[e^{h_2(iucot\alpha-\omega)} - e^{h_1(iucot\alpha-\omega)}\right]}{(iucot\alpha - \omega)e^{iuh_1 cot\alpha}}\right\} \tag{8}$$