Computers & Geosciences 62 (2014) 88–94

Contents lists available at ScienceDirect

Computers & Geosciences

journal homepage: www.elsevier.com/locate/cageo

# A novel modeling approach using arbitrary Lagrangian–Eulerian (ALE) method for the flow simulation in unconfined aquifers

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ARTICLE INFO

*Article history:*
Received 25 April 2013
Received in revised form
15 September 2013
Accepted 7 October 2013
Available online 15 October 2013

*Keywords:*
Arbitrary Lagrangian–Eulerian (ALE) method
Dupuit assumption
Free-surface problem
Groundwater flow simulation
Unconfined aquifer

ABSTRACT

The problem of groundwater flow in an unconfined aquifer, formulated as a free-surface problem, is solved numerically through a new approach by employing the arbitrary Lagrangian–Eulerian (ALE) method. The domain of interest is three dimensional or a two dimensional vertical cross-section of a phreatic zone of an aquifer, where the groundwater table is the upper boundary that is allowed to move. The ALE method allows capturing the location of the free-surface by transforming the moving domain to a fixed reference domain through arbitrary forced boundary conditions. The results of the verification runs of this new approach agree well with the known analytical solutions for aquifer characterization tests. Beside the comprehensive and accurate evaluation of the groundwater flow in the tested cases, the approach is also suitable for modeling complex situations. The implementation of our method for selected cases is illustrated by means of practically relevant examples.



## 1. Introduction

The characterization of subsurface flow in unconfined aquifers is a challenging task, given the difficulty of identifying the precise groundwater table (free-surface) positions (Castro-Orgaz and Giráldez, 2012). Analytical solutions can be obtained only when simplifying assumptions, such as Dupuit approximation, are introduced. It assumes (1) that the streamlines are horizontal for small inclinations of the line of seepage and (2) that the hydraulic gradient does not depend on depth and thus equals the slope of the free surface (see also Harr, 1962; Strack, 1989). Steady state unconfined flow and flow toward a well were described by Dupuit (1863) under these assumptions. Subsequently, analytical solutions were derived for unsteady unconfined flow (i.e., Boussinesq, 1904) and also for specific applications, such as evaluation of a pumping test (i.e., Neuman, 1972). However, all these analytical solutions assume a fixed water table condition even close to the pumping well, where a large drawdown is usually observed (Mishra and Kuhlman, 2013). The groundwater flow here is more complex and, therefore, the vertical flow cannot be neglected for many relevant applications (Bevan et al., 2005; Bunn et al., 2011; Dagan et al., 2009; Mishra and Kuhlman, 2013). Moreover, the inaccuracy of Dupuit's assumptions was demonstrated by Dagan et al. (2009), Desbarats and Bachu (1994), and Tartakovsky et al. (2000).

Another exact solution for two-dimensional steady flow is given by the so-called hodograph method, which depends on finding conformal mappings of the flow region in the physical plane (i.e., Bakker, 1997; Bear, 1972). However, the method also fails to yield an analytical solution, when the geometry of the boundaries becomes complicated (Bear, 1972). Consequently, the main limitation of analytical methods is that they are only available for relatively simple problems and are not flexible to describe complex application problems in detail (Bear and Verruijt, 1987).

Numerical methods, which have been developed since the 1960s (Fayers and Sheldon, 1962; Freeze and Witherspoon, 1966; Remson et al., 1965), have the advantage that they are applicable for more general situations when compared with analytical methods. In principle, two conceptual approaches are used: (1) the partially saturated or unsaturated–saturated flow approach, and (2) the fully saturated or water table/free-surface flow approach. On the one hand, the first approach considers the entire flow domain and solves the Richards equation above the water table and the groundwater flow equation at the saturated parts of an aquifer. The second approach, on the other hand, considers only the saturated parts of an aquifer and solves a free-surface problem. Models following the first approach are able to provide a more holistic and rigorous analysis of flow processes. However, the solution is usually hampered due to the difficulties of obtaining site-specific data for the unsaturated zone and due to the computational complications (Feddes et al., 2004; Knupp, 1996).

In view of these difficulties, the majority of groundwater models use the second approach, which takes the groundwater table as the upper moving boundary of the saturated zone and

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http://dx.doi.org/10.1016/j.cageo.2013.10.002

**Nomenclature**

| | |
|---|---|
| $\alpha$ | porous medium compressibility |
| $\beta$ | fluid compressibility |
| $\mu$ | fluid viscosity |
| $\rho$ | fluid density |
| $\varphi$ | porosity |
| $\boldsymbol{\Omega_x}$ | spatial domain |
| $\boldsymbol{\Omega_X}$ | reference domain |
| $v_r$ | ground water flow velocity |
| $d$ | drawdown in the well |
| $g$ | acceleration due to gravity |
| $h$ | hydraulic head |
| $j$ | mass flux |
| $\mathbf{k}$ | tensor for aquifer permeability |
| $p$ | pressure |
| $q$ | recharge/discharge |
| $r$ | radial distance from well |
| $t$ | time |
| $z$ | spatial axis in vertical direction |
| $D$ | well screen length |
| $\mathbf{F}$ | mesh deformation gradient |
| $K$ | hydraulic conductivity |
| $\mathbf{K}$ | tensor for hydraulic conductivity |
| $L$ | length of an aquifer |
| $Q$ | pumping rate |
| $S$ | storage coefficient |

relocates its position iteratively during the computation. One of the most common methods is the one employed in MODFLOW (Harbaugh et al., 2000; McDonald and Harbaugh, 1988). It solves either the confined or the unconfined groundwater flow equation depending on whether the grid is saturated or contains the water table. Limitations associated with this method, i.e., dry cells, numerical instability, and numerical errors, have been discussed in Banta (2006), Harbaugh et al. (2000), Keating and Zyvoloski (2009), Naff et al. (2003), and Zyvoloski and Vesselinov (2006). Another idea is to solve the free-surface problem with the location of the groundwater table as an additional unknown and adjust the mesh accordingly. Following this idea, Diersch (2009) introduced a so-called BASD (Best-Adaptation-to-Stratigraphic Data) method to trace the location of the free-surface with the software FEFLOW. However, this method requires a 3D model.

The main objective of this study is to provide a comprehensive simulation method that is able to capture the groundwater table position of an unconfined aquifer without having the above mentioned restrictions. In this paper, we follow the second conceptual approach and solve the free-surface problem only for steady state. The novel treatment of the free-surface is implemented making use of a generic mathematical algorithm, the arbitrary Lagrangian–Eulerian (ALE) method, in the flow simulation.

This paper is structured as follows. After reviewing the governing equations for groundwater flow in unconfined aquifer, the ALE method as well as its application in solving the free-surface problem is intensively discussed. This new method is tested by comparing the simulation results with the analytical solutions derived for classical cases. Furthermore, the advantages of this simulation method are presented through the relevant application cases. Finally, we summarize the paper and present the conclusions.

## 2. Governing equations

The governing equation that describes the flow of groundwater in saturated porous media is developed from the fundamental principle of mass conservation (continuity equation) and Darcy's Law. A detailed derivation of the governing equations is provided by Bear (1972) and Bear and Verruijt (1987). The governing equation for transient flow in unconfined aquifers is given as

$$(\alpha+\varphi\beta)\frac{\partial p}{\partial t}-q=\nabla\cdot\frac{\mathbf{k}}{\mu}\nabla(p+\rho g z) \tag{1}$$

where $\beta$, $\rho$ and $\mu$ denote the fluid properties compressibility, density and viscosity, respectively, $\alpha$ and $\varphi$ represent the porous medium compressibility and porosity, respectively, $q$ is the recharge or discharge, $\mathbf{k}$ indicates the tensor for aquifer permeability, $t$ is the time, $g$ is the acceleration due to gravity and $z$ denotes the spatial axis in vertical direction. The unknown variable of the differential equation is the pressure $p$.

Eq. (1) can also be stated in terms of hydraulic head $h$ that is defined as $h=z+p/\rho g$. For constant $\rho$ results

$$S\frac{\partial h}{\partial t}-q=\nabla\cdot\mathbf{K}\nabla h \tag{2}$$

where the tensor for hydraulic conductivity $\mathbf{K}$ is given by $\mathbf{K}=\mathbf{k}\rho g/\mu$, and the storage coefficient or storativity $S$ is presented by $S=\rho g(\alpha+\varphi\beta)$.

At steady state, Eq. (2) can be simplified to

$$\nabla\cdot\mathbf{K}\nabla h+q=0 \tag{3}$$

In the case where $q$ is not considered, Eq. (3) results in Laplace's equation.

## 3. The novel numerical approach

### 3.1. Model domain and groundwater flow equation

In the following examples, we solve the problems for steady state only. Two conditions need to be fulfilled at the free-surface: (1) zero pressure and (2) flow of groundwater recharge or discharge across the interface. Condition (1) is a Dirichlet condition for pressure where the atmospheric pressure is set to zero. Condition (2) is a Neumann condition for flow across the surface. In case that $q$ can be neglected in Eq. (3), it is of usual no-flow type. In the problem formulation, our strategy is to connect these two conditions by defining the flux condition for the flow equation and the pressure condition for the free-surface. To fulfill both conditions, the flow equation and the free-surface algorithm are coupled.

### 3.2. Tracing free-surface deformation with the ALE method

The ALE method is a hybrid description which uses a moving mesh to follow the change of a boundary simultaneously. More precisely, in this formulation, the coordinate system of the problem domain moves in a certain prescribed manner, which allows the computational mesh to follow or to deform together with the change of a free-surface. For more details about this algorithm one may refer to the articles by Donea et al. (2004). The ALE method has already been used for simulating general free-surface problems (e.g., Duarte et al., 2004; Maury, 1996; Pohjoranta and Tenno, 2011). To our knowledge, however, it has not yet been implemented for solving free-surface problems in groundwater flow simulations.

The basic principle of the ALE method is to superimpose the arbitrary deformed domain $\boldsymbol{\Omega_x}$, or spatial domain, and the