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Quadratic self-correlation: An improved method for computing local fractal dimension in remote sensing imagery ☆

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ABSTRACT

We present a new method for computing the local fractal dimension in remote sensing imagery. It is based on a novel way of estimating the quadratic self correlation (or 2D Hurst coefficient) of the pixel values. The method is thoroughly tested with a set of synthetic images an also with remote sensing imagery to assess the usefulness of the techniques for unsupervised image segmentation. We make a comparison with other estimators of the local fractal dimension. Quadratic self-correlation methods provide more accurate results with synthetic images, and also produce more robust and fit segmentations in remote sensing imagery. Even with very small computation windows, the methods prove to be able to detect borders and details precisely.

1. Introduction

Most spatial patterns of nature are so irregular and fragmented that Euclidean geometry fails to provide tools for the analysis of their form and shape. In contrast, fractal geometry is a useful formalism for describing and characterizing complex objects, specifically those arising in natural phenomena (Scholz and Mandelbrot, 1989). Fractal sets are those whose characteristic shape, variation of form, or degree of irregularity are scale invariant. It is possible to describe or characterize a fractal object or phenomena independently of the spatial or temporal scale of observation (Mandelbrot, 1983). Fractal dimensions (FDs) may be viewed as measures of the irregularity or heterogeneity of these sets. FDs are exponents that relate the self-affine invariance or statistically self-similarity of a given measure at different scales.

In mathematically defined sets, it is possible to find a deterministic measure using the self similarity FD, also called Hausdorff dimension (Falconer, 1989). However, when there is no mathematical definition of the set, this method is computationally unfeasible, and approximations are required. The usual methodology to obtain a self-similarity exponent for non-deterministic objects or models, in particular, the magnitude or level of a pixel in digital images (which may represent several physical measurements like luminance, radiance, reflectance, among others), consists in establishing a regression between the variation of any relevant feature of the set with respect to the measuring scale. For magnitudes like area, perimeter, or grey level variation, FDs based on local brightness variations, triangular prism (TP), and variogram are appropriate (Clarke, 1986; Mark and Aronson, 1984; Russ, 1993, 2011). On the other hand, for probability, occupation, entropy, or spectral FDs, other techniques are more adequate (Grassberger and Procaccia, 1983a,b).

The localized properties of variance (i.e., texture) frequently characterize the regions of an image more accurately than other features. For these reasons, fractal estimators constitute a suitable local descriptor for image segmentation for many applications. Among them, two major areas are remote sensing (e.g. landscape change detection and land use/cover classification) and medical imaging (e.g. tissue segmentation, malformations or tumor detection/localization, early diagnosis of neurodegenerative diseases) (Esteban et al., 2007; Reljin et al., 2000; Reljin and Reljin, 2002).

An efficient description of image texture is crucial for a successful classification or segmentation of images based on local spatial variations. Texture analysis refers to a class of mathematical

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procedures and models that characterize the spatial variations (Srinivasan and Shobha, 2008).

The many different properties that take an active part in texture description establish a wide variety of techniques for texture characterization. A review and classification of texture analysis algorithms can be found in Tuceryan and Jain (1998). It includes techniques based on *statistical methods*, in which the texture is characterized by statistical distribution of intensity values within a region of interest; *geometrical or structural methods*, where the texture is characterized by feature primitives and their spatial arrangements; *model-based methods*, in which a mathematical model describes the texture; and *signal-processing methods*, in which a set of filters with varying properties is used to compute texture features.

Grey level co-occurrence matrix (GLCM) (Haralick et al., 1973) is one of the most widely used statistical methods that estimates image properties related to second-order statistics. The GLCM of an image is the joint probability occurrence of grey levels i and j for two pixels with a spatial relationship defined in terms of distance d and angle θ. By changing these parameters, several matrices can be constructed and a variety of features may be extracted from them. Marrón (2012) focuses on how to apply texture operators based on the concept of co-occurrence matrix (energy, entropy and contrast) and fractal dimension (method of range, box counting and Hurst dimension); examples of segmentation over coast an city images illustrate the comparative study of the operators.

Autocorrelation measures provide a way to quantify the scale and spatial structure of images at different scales. Global measures like semivariance (Matheron, 1971), Geary's C (Geary, 1954), and Moran's I (Moran, 1948), summarize the spatial pattern across an entire image. On the other hand, local measures like G statistics (Getis and Ord, 1992), and LISA (Anselin, 1995), are necessary to understand the dominant contributors to the global metrics. This statistical analysis helps to overcome some masking effects of global measures by revealing areas of spatial non-stationarity, and thus may be used to identify a group of bright or dark pixels (‘hot spots’ or ‘cold spots’) that represent a spectral response from a homogeneous feature (Myint et al., 2007). Emerson et al. (1999) measured the global spatial autocorrelation of satellite imagery to observe the differing spatial structure of smooth and rough surfaces. Myint (2003) studied the effectiveness in extracting texture features or identifying different land-use and land-cover classes in remotely sensed images. The study compared spatial autocorrelation measures (Moran's I and Geary's C), three fractal approaches (isarithm, triangular prism, and variogram), and simple standard deviation and mean value of the selected features.

Although FDs are not always sufficient as a single feature for texture characterization, their invariance under geometric and brightness transformations is a required feature for robust unsupervised classification. The use of Gabor filters or other transformations of the original images as a preprocessing method for producing additional features is proposed in Chaundhuri and Sarkar (1995), and Dubuc et al. (1987). Kasparis et al. (2001) use a bank of N Gabor filters to process the original image and estimates the local fractal dimension (LFD) of them. With these LFDs, they obtain feature vectors of dimension N which are used for segmentation purposes. This methodology was tested on mosaics of texture samples from Brodatz (1966). The main drawback of this approach, or others based on invariant transformations, is the resulting computational complexity.

Different methods for FD estimation have been applied extensively in many disciplines. A survey of several commonly used methods for estimating the fractal dimension and their applications to remote sensing problems can be found in Sun et al. (2006). The paper presents a description of six computational methods (triangular prism, differential box counting, variogram, isarithm, robust fractal estimator and power spectrum).

Triangular prism (TP) (Clarke, 1986) has become one of the most often used methods with remote-sensing images (Sun et al., 2006; Ju and Lam, 2009; and references therein). Clarke's original algorithm estimates the surface's FD as $2-B$ where B is the slope derived from the log–log regression between the total prism surface area and the step size squared. This method underestimates the FD, whereas a modified TP using the step size leads to experimentally more precise results (Zhao, 2001).

In Quattrochi et al. (1997) and Lam et al. (1998), the image characterization and modeling system (ICAMS) is introduced. ICAMS provides the ability to calculate the FD of images using isarithm, variogram and the modified TP methods (Goodchild, 1980; Mark and Aronson, 1984). All of them were compared in Lam et al. (2002). In Zhou and Lam (2009), five FD estimators were compared (probability, variation, and the three aforementioned) using synthetic surfaces generated with three surface generation algorithms. In both works, the same conclusions were reached with respect to the FD estimators: the modified TP and the isarithm algorithms have the lowest RMSE and standard deviation. The authors mention also that the surfaces generated using the random midpoint displacement (RMD) algorithm were more congruent with the estimated FD than the surfaces generated with shear displacement or with Fourier filtering.

An adjustment of the improved TP method for extending its applications in local measurements was introduced in Ju and Lam (2009). The algorithm calculates the FD within a local window (local FD or LFD) aiming at image segmentation using LFD as a feature. The new algorithm, called divisor-step (DS) due to the sampling method used to improve the window coverage, was found to be more robust and accurate than other sampling alternatives like the conventional geometric-step (with fixed and varying coverage) and the arithmetic-step (Emerson et al., 2005; Lam et al., 1998; Quattrochi et al., 1997).

Another technique for LFD estimation can be derived from the Hurst self-correlation exponent H, which characterizes the expected range of variation ΔV of a function within a neighborhood of size Δr, enabling a fit of the form $\Delta V \sim (\Delta r)^H$. It is possible to show that the resulting FD of such a function is $D=2-H$ (Mandelbrot and Van Ness, 1968).

For digital images, computing H for a given pixel p entails the regression (in log–log space) of the grey level variation ΔV within a window around p using disks of decreasing radius r. The larger the window we use, the more accurate estimation we obtain, but the computational cost increases and the spatial accuracy decreases (i.e. locating borders). In the case of surfaces, the relation between D and H is $D=3-H$, where $0 \le H \le 1$ (Russ, 1993). As will be shown later, this simple way of generalizing H estimation to 2D sets using linear self-correlation (LSC) is prone to produce meaningless results.

An enhanced method to estimate the self-correlation coefficient in surfaces was introduced in Silvetti and Delrieux (2007), where a *quadratic* self-correlation (QSC) coefficient is evaluated leading to more precise LFD estimations with respect to the LSC, at the expense of a higher computational cost for the same window size. In this work we present normalized integral fit (NIF), an alternative to least square fit (LSF) that allows to compute a distinct quadratic exponent. The novelty is based on the way of evaluating the assessment of ΔV within a disc of radius r, and how to weight the different ΔV values in the computation of the quadratic self-correlation coefficient. The resulting LFD estimation shows to be more robust and precise than other methods. We compare our previous and new methods (QSC-LSF and QSC-NIF) against LSC and DS (precision, accuracy, invariance under geometric and brightness transformations) both with synthetic and