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Three-dimensional damage analysis by the scaled boundary finite element method

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ABSTRACT

A novel and effective approach within the framework of the scaled boundary finite element method (SBFEM) is proposed for the damage analysis of structures in three dimensions. The integral-type nonlocal model is extended to SBFEM to eliminate the mesh sensitivity concerning the strain localization. In order to reduce the number of degrees of freedoms (DOFs), an automatic mesh generation algorithm using octree decomposition is employed to refine the localized damage process zone (DPZ), but no extra effort is required to deal with hanging nodes existing between adjacent subdomains with different sizes. A double-notched tension beam is simulated with two different meshes to illustrate the mesh-independence. Three benchmarks are modelled to further verify the effectiveness and robustness of the proposed approach. It is shown that the proposed computational approach is capable of accurately capturing the damage evolution under complicated boundary conditions, and the results agree well with the experimental observations and prior numerical simulations reported in the literatures.

1. Introduction

Three-dimensional fracture modelling is a challenge topic in numerical simulation. Various types of numerical method, such as Finite Element Method (FEM) [1], Extended Finite Element Method (XFEM) [2], Meshless/Meshfree Method (MM) [3,4], and Dual-Horizon Peridynamics (DH-PD) [5], have been developed to simulate the crack propagation with or without remeshing procedure concerning the crack faces variation, based on fracture mechanics (FM) and continuum damage mechanics (CDM) respectively.

CDM, initiated in the context of creep rupture [6], is widely used to simulate the diffuse fracture process at both macroscopic [7,8] and mesoscopic level [9]. CDM models use internal variables to describe the gradual loss of material integrity due to the propagation and coalescence of microdefects. Compared with discrete crack models, CDM has become a very competitive approach to simulate the progressive failure of structures, which meticulously handles the strong discontinuity of crack without the requirement of remeshing.

Assuming that the stress at a specified point only depends on the state variables at that point, a local damage model exhibits an extreme sensitivity to the fineness and orientation of the spatial discretization in a mesh-based formulation [7,9]. Such a pathological phenomenon is caused by the fact that the mathematical description becomes ill-conditioned at a certain level of accumulated damage. To overcome the deficiencies of the local damage model, several types of nonlocal approaches have been proposed during the last decades. Among many innovative developments, the integral-type nonlocal model [7,10] and the gradient-type nonlocal model [11,12] are two of the most prevalent model. The integral-type nonlocal model involves a spatial smoothing function to average the state variable of a point in a certain range of internal length (also called characteristic length), whereas the gradient-type nonlocal model takes the field in the immediate vicinity of the point into account by enriching the local constitutive relations with gradients of some state variables. Among the gradient-type formulations, the implicit gradient enhancement [13] is found to be more effective and suitable for numerical implementation than the explicit version. Many works involving nonlocal damage simulations have been reported, but only a few of them [14,15] covered 3D problems mainly because of the unacceptable computational cost, especially on the solution of large nonlinear equilibrium equations [15].

The scaled boundary finite element method (SBFEM) is a semi-analytical method initiated by Wolf and Song in 1990s [16] for the solution of wave propagation problems in unbounded domain. The SBFEM combines some of the advantages of both the finite element

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method (FEM) and the boundary element method (BEM). In SBFEM, only the boundary is required to be discretised and the dimensions of the problem can be reduced by one, which in turn reduces the data preparation and computational effort. But the SBFEM does not require a fundamental solution as needed in BEM. The ability to exactly satisfy radiation conditions at infinity makes this method particularly suitable for modelling unbounded media [17]. Owing to its capability of obtaining semi-analytical stress intensity factors, the SBFEM is also an attractive method for crack initialisation [18,19] and crack propagation modelling [20,21]. Furthermore, the SBFEM is an extremely versatile approach in terms of meshing, since: (1) there is no limitation of the number of edges and vertices used for one subdomain, therefore, a series of polygonal subdomains with arbitrary shapes and sizes constructed by topological algorithms, e.g. Delaunay triangulation, can be used to discretise a domain in 2D with complex geometry profiles [22]. The SBFEM can also been combined with the isogeometric analysis to further increase the accuracy of the solution with exact geometric representation [23]; (2) Without any extra effort to deal with hanging nodes existing between adjacent subdomains with different sizes, quadtree structure in 2D and octree structure in 3D can be employed to discretise the domain with multi-level sizes of subdomains [24,25]. Consequently, the refinement of local regions can be controlled by certain parameters, and the transition between large-size subdomains and small-size ones is fast and straightforward.

In this contribution we propose a three-dimensional nonlocal approach for the damage analysis within the framework of SBFEM. The integral-type nonlocal model combined with the isotropic damage constitutive law is employed and extended to the scaled boundary formulation in 3D. Some inherent advantages of SBFEM are exploited for nonlocal damage modelling, including:

(I) In SBFEM, a problem domain can be discretised by a set of subdomains with various sizes and different number of edges. For damage analysis, in order to reduce the degrees of freedom (DOFs), a multi-level mesh in octree structure as illustrated in Section 4, is strongly recommended to mesh the domain with localized damage process zone (DPZ). Without any effort to deal with hanging nodes, this mesh generation algorithm is completely automatic and notably efficient. It should be pointed out that, octree structure is also adopted in FEM owing to its ability to transition between different cell sizes efficiently [26]. However, the displacement incompatibility is inevitably introduced within FEM by the hanging nodes presented between adjacent cells with different sizes. In order to deal with such a problem, complex shape functions are required for cells with hanging nodes [27] or further discretise of the cell with tetrahedralised cells is necessary [28];

(II) For damage simulation, the size of subdomains used in DPZ is especially small (typically around one-fourth to one-third of the characteristic length) and the numerical accuracy is mostly dependent on the damage estimation in DPZ. From the numerical point of view, as the mesh is refined, the results of simulation would converge to the accurate solution. It is reasonable to assume that the severity of damage is uniform in one subdomain, and only the strain at the scaling centre of the subdomain is used to compute the internal variable. Consequently, computational efforts can be considerably saved;

(III) Since the strain modes for each subdomain is only depending on the geometry of the subdomain, it can be computed beforehand and utilized at each load step to obtain the strain at an arbitrary point within the domain combined with updated integral constants (see Section 5);

(IV) For small deformation situations, the location of each subdomain is assumed to be unchanged during damage process, thus the weight function can also be calculated beforehand and utilized to smooth the internal variable in each load step.

The paper is organized as follows. The constitutive relation, damage evolution law and integral-type nonlocal model are introduced in Section 2. Section 3 presents a basic conception, equation and solution of SBFEM for elastostatics in 3D. In Section 4, the automatic mesh generation algorithm through octree decomposition is illustrated. Section 5 interprets the details of damage formulation using SBFEM. Four benchmark simulations are given in Section 6. Conclusions are presented in Section 7.

2. Damage model for concrete

2.1. Constitutive law

For isotropic damage model, the following equation is used to describe the stress-strain relation

$$\boldsymbol{\sigma} = (1-\omega)\mathbf{D}\boldsymbol{\varepsilon} \tag{1}$$

with the strain matrix $\boldsymbol{\varepsilon}$, the stress matrix $\boldsymbol{\sigma}$ and the elasticity matrix $\mathbf{D}$. ω is the damage variable which ranges from 0 to 1 at complete damage.

In the damage theory, it is natural to work in the strain space and, therefore, the arguments of the loading function include the strain $\boldsymbol{\varepsilon}$ and an internal variable κ, which is controlling the evolution of damage [29]. The loading function usually has the form

$$f(\boldsymbol{\varepsilon},\kappa) = \tilde{\varepsilon}(\boldsymbol{\varepsilon}) - \kappa \leqslant 0. \tag{2}$$

where $\tilde{\varepsilon}$ is a certain scalar named the equivalent strain to measure the strain level, κ is an internal variable that corresponds to the maximum level of equivalent strain ever reached in the previous history of the material up to the current state.

The internal variable κ starts at a damage threshold level κ_0 and is updated by the requirement that during damage growth $f = 0$, whereas in unloading stage $f < 0$ and $\dot{\kappa} = 0$. Damage growth occurs according to an evolution law $\omega = \omega(\kappa)$, which can be identified from the uniaxial stress-strain curve. The loading-unloading conditions of inelastic constitutive models are often formalised using the Karush-Kuhn-Tucker conditions [14]:

$$f \leqslant 0, \quad \dot{\kappa} \geqslant 0, \quad f\dot{\kappa} = 0. \tag{3}$$

2.2. Evolution of damage

The damage model described in the previous section contains two relations which are specific for a material, i.e. the damage evolution law $\omega(\kappa)$ and the equivalent strain definition. Two types of evolution law are widely used for damage model. The first one is linear softening model described as

$$\omega(\kappa) = \begin{cases} 0, & \text{if } \kappa \leqslant \varepsilon_0, \\ \frac{\varepsilon_f}{\varepsilon_f-\varepsilon_0}\left(1-\frac{\varepsilon_0}{\kappa}\right) & \text{if } \varepsilon_0 < \kappa < \varepsilon_f, \\ 1, & \text{if } \kappa > \varepsilon_f. \end{cases} \tag{4}$$

where ε_0 is the threshold of damage, and ε_f is a parameter affecting the ductility of the response and related to the fracture energy.

The second one is exponential softening model which is defined as

$$\omega(\kappa) = \begin{cases} 0, & \text{if } \kappa \leqslant \varepsilon_0, \\ 1-\frac{\varepsilon_0}{\kappa}\exp\left(-\frac{\kappa-\varepsilon_0}{\varepsilon_f-\varepsilon_0}\right), & \text{if } \kappa > \varepsilon_0. \end{cases} \tag{5}$$