Computers and Structures xxx (2018) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Free vibration reanalysis of structures with added degrees of freedom

S.P. Zheng [a], B.S. Wu [b,*], Z.G. Li [c]

[a] College of Construction Engineering, Jilin University, Changchun 130026, PR China
[b] School of Electro-Mechanical Engineering, Guangdong University of Technology, Guangzhou 510006, PR China
[c] School of Mathematics, Jilin University, Changchun 130012, PR China

ARTICLE INFO

Article history:
Received 27 March 2018
Accepted 6 June 2018
Available online xxxx

Keywords:
Free vibration reanalysis
Added degrees of freedom
Shifting
Modified initial analysis system
Block combined approximations

ABSTRACT

This paper deals with free vibration reanalysis for structural topological modifications with added degrees of freedom. A modified initial analysis system (MIAS) is first constructed to include the newly added degrees of freedom. The MIAS is then used to analyze the modified structure, keeping the number of degrees of freedom unchanged. The block combined approximation method with shifting is finally utilized to calculate multiple eigensolutions of the modified structure. The implementation of the approach involves only LDL^{T} factorizations of shifted sub-stiffness matrices corresponding to the newly added degrees of freedom. The proposed method consists of matrix–matrix operations. These operations can be implemented by using Level-3 Basic Linear Algebra Subprograms, the execution efficiency is thus enhanced. Numerical examples are given to demonstrate the effectiveness of the proposed method and the accuracy of the approximate solutions.

1. Introduction

The process of structural design is an iterative one. In general, a structure may be gradually modified during the process of design or optimization until a satisfactory design is achieved. For large-scale problems, especially, dynamic problems, repeated analyses involve high computational costs. Therefore, it is necessary to develop a fast reanalysis method.

Free vibration reanalysis is intended to efficiently compute the structural eigensolutions for various changes in design without solving the full set of modified analysis equations. Such a solution procedure usually uses the eigensolutions of the initial structure. In design or optimization, structural topology may change. Typical cases of topological modifications can be generally classified as follows [1]:

(1) Deletion and addition of elements, where the number of degrees of freedom (DOFs) is unchanged.
(2) Deletion and addition of elements, and deletion of some original nodes, where the number of DOFs is decreased.
(3) Deletion and addition of elements, and addition of some new nodes, where the number of DOFs is increased.

In previous studies, static reanalysis techniques have been developed for all the above cases of topological modifications [1–10]. For the case (1) where the number of DOFs keeps unchanged, various methods have been proposed for free vibration reanalysis. These methods include combined approximation (CA) [5], the substructuring technique associated to locally modified regular structures [11], the frequency-shift CA [12], homotopy perturbation and projection techniques [13], the modified CA [14], the block combined approximation with shifting (BCAS) [15], and so on. For the case (2) where the number of DOFs is decreased, solution procedures described in [5,11–15] can be used by eliminating the corresponding DOFs in modes of the initial structure. For the case (3) of structural topological modification where the nodes of the original structure are a subset of the nodes of the modified structure, the number of DOFs of the analysis model will increase. The free vibration reanalysis for such a modification is particularly challenging. Limited studies have been published on free vibration reanalysis for the case. A free vibration reanalysis procedure for topological modifications with Ritz analysis was presented in [16]. Based on modal analyses of the initial structure, expanded basis vectors and one-step subspace iteration were utilized to obtain approximate eigensolutions of the modified structure. Since the expanded basis vectors were formed by a transformation which neglected the inertia effect of the added DOFs, the accuracy of the extracted eigensolutions deteriorated significantly with the increase of the required eigensolutions. Afterward, a free vibration reanalysis method through improving Ritz basis vectors using

* Corresponding author.
E-mail address: wubs@gdut.edu.cn (B.S. Wu).

https://doi.org/10.1016/j.compstruc.2018.06.006
0045-7949/

matrix perturbation technique was proposed in [17]. The newly added DOFs were condensed by means of Guyan reduction. Rong et al. [18] presented a free vibration reanalysis method for structures subjected to topological modifications. The expanded basis vectors were formed by iterative perturbation method based on results of the initial structure. However, when the inverse of dynamic stiffness matrix was calculated by the Neumann series [18], it was not always convergent. Yang et al. [19] put forward a method for free vibration reanalysis of topological modifications of structures. The accuracy of the solutions of the original DOFs was improved by using the extended Kirsch method, and the newly added DOFs were linked to the original DOFs by means of the dynamic reduction. Kaveh and Fazli [20] presented a method for calculation of eigensolutions of Laplacian matrix corresponding to a graph composed of two non-overlapping graph products by using the shifted inverse iteration technique. Owing to the very fast methods available for eigensolutions of graph products, this method is more efficient compared with the standard shifted inverse iteration. For modal reanalysis method of structural simultaneous and multiple type modifications, an approximate method, which combined the dynamic condensation, independent and coupling mass orthogonalization with the Rayleigh–Ritz analysis was proposed by He et al. [21]. However, the above-mentioned methods can only be used to obtain low-order modes of the modified structure. Except [20], no shifting strategy is used.

In this paper, a modified initial analysis system (MIAS) is firstly presented for free vibration reanalysis of topological modifications with added DOFs, such that the added DOFs are included in the analysis model. The block combined approximation with shifting (BCAS) [15] is then applied to compute multiple eigensolutions of the modified structure. The proposed method needs only LDL^{T} factorizations of the shifted sub-stiffness matrices corresponding to the added DOFs. For improving computational efficiency, Basic Linear Algebra Subprograms (BLAS) are employed. The BLAS are routines that provide standard building blocks for performing basic vector and matrix operations. They are widely used as a building block in higher-level mathematical programming languages and libraries, including LINPACK, LAPACK and so on. The Level-1 BLAS perform scalar, vector and vector-vector operations; the Level-2 BLAS perform matrix-vector operations; the Level-3 BLAS perform matrix-matrix operations and thereby its higher performance is achieved. The matrix-matrix operations in the proposed method are based on the Level-3 BLAS, the computational efficiency is significantly improved [22]. Three numerical examples are presented to illustrate the effectiveness of the proposed method and the accuracy of the approximate solutions.

2. Problem statement

Consider an original structure with $m \times m$ stiffness matrix $\mathbf{K}_0$ and mass matrix $\mathbf{M}_0$, respectively. Both of them are sparse and symmetric, and $\mathbf{M}_0$ is positive definite. The corresponding p lowest eigensolutions can be computed by solving the following eigenproblem via subspace iteration (SI) or Lanczos algorithms

$$\mathbf{K}_0\boldsymbol{\Phi}_0 = \mathbf{M}_0\boldsymbol{\Phi}_0\boldsymbol{\Lambda}_0 \tag{1}$$

where $\boldsymbol{\Lambda}_0 = diag(\lambda_{01}, \lambda_{02}, \ldots, \lambda_{0p})$ and $\boldsymbol{\Phi}_0 = [\boldsymbol{\varphi}_{01}, \boldsymbol{\varphi}_{02}, \ldots, \boldsymbol{\varphi}_{0p}]$ represent the matrices of p eigenvalues and the corresponding eigenvectors of the original structure, respectively.

It is efficient to use the shifting technique to improve the convergence rate for higher modes in the free vibration analysis of the initial structure [23–25]. Define the generalized eigenproblem with shifting μ

$$(\mathbf{K}_0 - \mu\mathbf{M}_0)\boldsymbol{\Phi}_0 = \mathbf{M}_0\boldsymbol{\Phi}_0\widehat{\boldsymbol{\Lambda}} \tag{2}$$

where $\widehat{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda}_0 - \mu\mathbf{I}_p$ and $\mathbf{I}_p$ is unit matrix of order p. Two eigenproblems in Eqs. (1) and (2) have the same eigenvectors. By use of the SI method with shifting or shifted Lanczos algorithms, the modes will converge to the ones having the smallest shifted eigenvalues. For more details, we refer readers to [23–25]. As a result, the $\mathrm{LDL^T}$ factorized form of initial stiffness matrix $\mathbf{K}_0$ and those of a series of shifted stiffness matrices $\mathbf{K}_0 - \mu_{0j}\mathbf{M}_0$ $(j = 1, 2, \ldots, k)$ can been given from the initial analysis as follows

$$\mathbf{K}_0 = \mathbf{L}_0\mathbf{D}_0\mathbf{L}_0^T \tag{3}$$

$$\mathbf{K}_0 - \mu_{0j}\mathbf{M}_0 = \mathbf{L}_{0j}\mathbf{D}_{0j}\mathbf{L}_{0j}^T, \quad j = 1, 2, \ldots, k \tag{4}$$

where $\mathbf{L}_0$ and $\mathbf{L}_{0j}$ $(j = 1, 2, \ldots, k)$ are lower triangular matrices with unit elements on their diagonals, $\mathbf{D}_0$ and $\mathbf{D}_{0j}$ $(j = 1, 2, \ldots, k)$ are diagonal matrices, and $k + 1$ is number of these LDL^{T} factorizations.

Suppose that, for a topological modification, some new nodes are added to the original structure, the number of DOFs of the modified structure is then increased. This will result in that the number of the analysis equations as well as the size of the stiffness and mass matrices are enlarged. Denote the stiffness matrix $\mathbf{K}_M$ and mass matrix $\mathbf{M}_M$ of the topologically modified structure in the following partitioned form, respectively.

$$\mathbf{K}_M = \begin{bmatrix} \mathbf{K}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \Delta\mathbf{K}_{mm} & \Delta\mathbf{K}_{mn} \\ \Delta\mathbf{K}_{nm} & \Delta\mathbf{K}_{nn} \end{bmatrix} \tag{5}$$

and

$$\mathbf{M}_M = \begin{bmatrix} \mathbf{M}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \Delta\mathbf{M}_{mm} & \Delta\mathbf{M}_{mn} \\ \Delta\mathbf{M}_{nm} & \Delta\mathbf{M}_{nn} \end{bmatrix} \tag{6}$$

where n is the number of DOFs resulting from the newly added nodes, $\Delta\mathbf{K}_{nn}$ and $\Delta\mathbf{M}_{nn}$ are the stiffness and mass matrices for the newly added nodes and elements, $\Delta\mathbf{K}_{nm} = \Delta\mathbf{K}_{mn}^T$ and $\Delta\mathbf{M}_{nm} = \Delta\mathbf{M}_{mn}^T$ describe the stiffness and mass matrices expressing the connection of the original DOFs with the newly added DOFs, $\Delta\mathbf{K}_{mm}$ and $\Delta\mathbf{M}_{mm}$ are the changes of the stiffness and mass matrices of the original DOFs due to the newly added DOFs.

The free vibration eigenproblem of the modified structure can be expressed as

$$\mathbf{K}_M\boldsymbol{\Phi}_M = \mathbf{M}_M\boldsymbol{\Phi}_M\boldsymbol{\Lambda}_M \tag{7}$$

where $\boldsymbol{\Lambda}_M = diag(\lambda_{M1}, \lambda_{M2}, \ldots, \lambda_{Mp})$ and $\boldsymbol{\Phi}_M$ are the matrices of the eigenvalues and eigenvectors, respectively, to be determined. The matrix $\boldsymbol{\Phi}_M$ of eigenvectors can be expressed as

$$\boldsymbol{\Phi}_M = \begin{bmatrix} \mathbf{U}_m \\ \mathbf{V}_n \end{bmatrix} \tag{8}$$

where $\mathbf{U}_m$ and $\mathbf{V}_n$ represent the matrices of the eigenvectors corresponding to the original DOFs and the newly added DOFs, respectively.

The purpose of free vibration reanalysis is to efficiently obtain the approximate p eigensolutions of Eq. (7) for the modified structure without directly solving the complete eigenproblem.

3. Modified initial analysis systems

Due to adding of some joints to the original structure, the number of DOFs in the modified structure is increased and the size of eigenproblem matrices is expanded. As a result, it is necessary to construct a MIAS so that the number of DOFs keeps unchanged in the analysis model.

Based on Eq. (1), the stiffness and mass matrices $\widetilde{\mathbf{K}}_0$ and $\widetilde{\mathbf{M}}_0$ in the MIAS can be established as follows.

Please cite this article in press as: Zheng SP et al. Free vibration reanalysis of structures with added degrees of freedom. Comput Struct (2018), https://doi.org/10.1016/j.compstruc.2018.06.006