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An improved explicit time integration method for linear and nonlinear structural dynamics

Wooram Kim *, Jin Ho Lee

Department of Mechanical Engineering, Korea Army Academy at Yeongcheon, Yeongcheon-si, Gyeongsangbuk-do 38900, Republic of Korea

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ABSTRACT

In this article, a new explicit time integration method is developed to analyze linear and nonlinear problems of structural dynamics. Like recently developed explicit time integration methods, the new explicit method can also control the amount of numerical dissipation in the high frequency range. The method is explicit in the presence of the damping matrix, if the mass matrix is diagonal. Due to the unconventional approximations of the displacement vector, the new method does not require evaluation of the initial acceleration vector and other acceleration vectors. Linear and nonlinear problems of structural dynamics can be tackled in a consistent manner, and iterative solution finding procedures are not required. Various illustrative problems are used to investigate improved performance of the new explicit method.

1. Introduction

Recently, numerous implicit and explicit time integration methods were proposed for effective analyses of structural dynamics. Many of the recently developed time integration methods possess controllable numerical dissipations which are useful for eliminating the spurious high frequency mode in numerical solutions [1,2]. In general, implicit methods are unconditionally stable when they are applied to linear problems, while explicit methods are only conditionally stable. Due to this fact, dissipative implicit methods can be used for the high frequency filtering by adopting considerably large time steps, and numerical dissipations in explicit methods are usually used to improve quality of numerical solutions in wave propagation and impact problems where small time steps are required.

Other than stability conditions, the biggest difference between implicit and explicit methods can be found in equation solving procedures. In implicit methods, the displacement and velocity vectors of current time step are expressed in terms of both unknown properties of current time step and known properties of previous time steps. Naturally, implicit methods require factorizations of the effective stiffness matrices which are not diagonal to solve the fully discrete equations.

In linear analyses, factorization of the effective stiffness matrix is required only once, if the factorized effective stiffness matrix is stored in additionally allocated memories and reused for next time steps. In nonlinear analyses, however, the internal force vectors and the stiffness matrices are often functions of unknown displacement and velocity vectors of current time. Due to this reason, construction and factorization of the effective stiffness matrix are inevitable in each time step for implicit methods, and each time step accompanies several times of iterations to obtain converged nonlinear solutions. In large and complex nonlinear systems, this may seriously limit solution refinements which can be done by decreasing sizes of time steps, because factorization of a big effective stiffness matrix requires huge computational resources. Details regarding recent development of implicit methods and their computational aspects can be found in Refs. [2–5].

On the other hand, factorization of any matrices is not required in explicit methods if the mass matrix is diagonal. Due to this fact, explicit methods require much less computational effort to advance a time step compared with implicit methods. In nonlinear problems, the entries of the mass matrix are usually given as constants, while the damping matrix and the internal force vector are functions of the current displacement and velocity vectors. Even in these situations, well designed explicit methods does not require any factorizations of matrices, if the mass matrix is diagonal. Even when the mass matrix is not diagonal, factorization of the mass matrix is required only once, if the factorized mass matrix is stored in additionally allocated memories and reused for next time steps. Thus, explicit methods may be more efficient for analyses of large nonlinear systems that require very small sizes of time steps for a long duration of time.

* Corresponding author.
E-mail address: kim.wooram@yahoo.com (W. Kim).

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The Newmark method is one of the most broadly used non-dissipative second-order accurate implicit method [6] for structural dynamic problems. After the introduction of the Newmark method, numerous improved implicit methods were developed based on it. One of the most famous methods developed based on the Newmark method is the generalized-α method [7]. The generalized-α method of Chung and Hulbert can control numerical dissipations of the high frequency limit in a simple and practical manner. Recently, some implicit methods were developed based on the time finite element approach. The collocation composite time integration method of Kim and Reddy [5] and the generalized composite method of Kim and Choi [4] are recently proposed second-order accurate implicit methods based on the time finite element method.

Good explicit methods can also provide very accurate numerical solutions for very complicated nonlinear problems with much less computational effort. The 4th-order Runge-Kutta method (the RK4 method) and the central difference method (the CD method) are standard explicit methods which can be used for structural dynamics. The CL method of Chung and Lee provided acceptably accurate solutions for the elastic spring-mass nonlinear pendulum problem [8]. The HC method of Hulbert and Chung [9], the TW method of Tchamwa and Wielgosz [10,11], and the NB method of Noh and Bathe [12] was developed for the analyses of wave propagations and impact problems. Recently, the Soares method [13] was developed based on the weighted residual approach.

The explicit methods mentioned above have their own advantages and disadvantages. The RK4 method was originally considered for general first-order ordinary differential equations. By rearranging equations of structural dynamics as proper first-order forms, structural dynamics can also be analyzed with the RK4 method, but this method requires more than four times of computational cost compared with the CD method. The RK4 method provides a considerably large amount of numerical dissipation (the minimum spectral radius is about 0.5) in the high frequency range when applied to the second-order linear single-degree-of-freedom problem. The CD method is the simplest non-dissipative explicit method. The CD method is provided as a standard time integration method in many software packages, but the CD method becomes implicit in the presence of the viscous damping terms.

The CL method can maintain explicitness in the presence of viscous damping terms. However, the amplification matrix of the CL method has the spurious eigenvalue that may seriously influence quality of solutions when large time steps are used. The minimum spectral radius of the most dissipative case of the CL method is about 0.52 when $\beta = 28/27$ is used, and only less dissipative cases are included in the CL method. Unlike the CL method, the HC method can include a full range of dissipative cases, however, the non-dissipative case of the HC method becomes *unconditionally unstable* for any choices of time steps in the presence of viscous damping terms. The amplification matrix of the HC method also has the spurious root.

The TW method does not require computation of the initial acceleration vector, and the method is very effective for wave propagation problems. On the other hand, all dissipative cases of the TW method are only first-order accurate, thus this method is not suitable for long term analyses. The Soares method is the only self starting method among the explicit methods mentioned above. It has dissipation control capability and provides improved accuracy and extended stability limit for linear problems. However the Soares method cannot be used for nonlinear analyses because it directly manipulated the linear structural dynamics equations in a weighted residual sense. The Soares method becomes only first order accurate in the presence of physical damping terms, and requires integral evaluation of the external force vector.

The NB method is probably the best performing second-order accurate explicit method among the explicit methods mentioned above. The NB method is developed based on the strategy similar to the strategy used in the implicit Bathe method where two sub-steps were combined to form one complete method. The NB and Soares method share almost identical spectral characteristics for the linear undamped single-degree-of-freedom problem. However, the NB method can be applied to nonlinear analyses in a consistent manner, while the Soares method cannot. Unlike the Soares method, the NB method requires computation of the initial acceleration vector, and the amplification matrix of the NB method always has spurious root when viscous damping terms are included.

In this work, we propose a new second-order accurate explicit time integration method to tackle variety of linear and nonlinear problems of structural dynamics in a consistent way. In designing the new explicit method, we wish to accommodate the preferable attributes of the existing explicit methods and exclude undesirable attributes by manipulating proper numerical techniques and methods. To this end, we consider unconventional interpolating techniques, effective residual minimizing procedures, and effective computational structures of recently developed explicit methods.

Discussions of this paper will mainly focus on the development and analysis of the proposed algorithms. Through the interpolating techniques used herein, we wish to exclude the spurious root of the amplification matrix of the proposed explicit method. By adopting unique computational structures of the Noh and Bathe method, we wish to achieve extended stability limit and improved spectral characteristics in the proposed method. By using the collocation approach for the time discretization, we also expect that the proposed method will be applicable to both linear and nonlinear problems in a consistent manner. In addition to these improvements, we also wish to eliminate computation of the acceleration vectors (including the initial acceleration vector) in our explicit method, which is already realized in the Soares method. Simple and illustrative linear and nonlinear single- and multi-degree-of-freedom benchmark problems will be used to investigate the linear and nonlinear performances of the proposed explicit method.

2. An explicit time integration method

The influence of the spurious root of time integration methods was studied in Ref. [14]. Even though the influence of the spurious root is not that huge in some of recent time integration methods as explained in Refs. [8,14], its presence is not completely acceptable in a mathematical view point. At least, it should be minimized to achieve good accuracy for the important low frequency modes. Here, the time finite element method [15–18] based unconventional Hermite type interpolating techniques are employed as a remedy for this problem. Many of improved methods, such as the HC, CL and NB methods [8,9,12], have the spurious roots, and initial and other acceleration vectors should always be computed and stored in each time step.

A practical way of designing explicit time integration methods without a spurious root is to exclude time nodal acceleration vectors from the time approximations of the displacement vector. This can be done by using proper Hermite type interpolation functions which are associated with time nodal displacement and velocity vectors for the approximation of the displacement vector [2,19]. Then, the approximated displacement vector, and the first and second time derivatives of the approximated displacement vector can be substituted into the structural dynamics equations to obtain approximated structural dynamics equations. The time discretization can be completed by evaluating the approximated structural dynamics equations at a certain point of time in a collocation

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