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A global-local approach for the elastoplastic analysis of compact and thin-walled structures via refined models

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ABSTRACT

A computationally efficient framework has been developed for the elastoplastic analysis of compact and thin-walled structures using a combination of global-local techniques and refined beam models. The theory of the Carrera Unified Formulation (CUF) and its application to physically nonlinear problems are discussed. Higher-order models derived using Taylor and Lagrange expansions have been used to model the structure, and the elastoplastic behavior is described by a von Mises constitutive model with isotropic work hardening. Comparisons are made between classical and higher-order models regarding the deformations in the nonlinear regime, which highlight the capabilities of the latter in accurately predicting the elastoplastic behavior. The concept of global-local analysis is introduced, and two versions are presented - the first where physical nonlinearity is considered for both the global and local analyses, and the second where nonlinearity is considered only for the local analysis. The second version results in reasonably accurate results compared to a full 3D finite element analysis, with a twofold reduction in the number of degrees of freedom.

1. Introduction

Metallic structures are ubiquitous in various fields of engineering, and it is thus important to understand their mechanical behavior to optimize the design and predict failure. These structures typically undergo plastic deformation when loaded past the yield point, which necessitates a nonlinear analysis to determine the elastoplastic behavior. Numerical simulation is an important tool for such an analysis and is usually performed within the framework of the Finite Element Method. However, accurate stress fields are required when nonlinearities are involved, which often means that a 3D finite element analysis has to be performed. Such 3D simulations can be computationally very expensive, especially for the case of complex slender structures such as thin-walled beams. Significant efforts have therefore been exerted over the past few decades to find suitable alternatives to full 3D analysis. A starting point to achieve this is using analytical models, whereby intensive numerical computations can be avoided. An analytical formulation for inelastic beams was proposed by Timoshenko and Gere [1], whose validity was limited to doubly symmetric cross-sections and neglected shear deformations. An analytical solution to the elastoplastic bending of beams was reported by Štok, for the case of rectangular cross-sections [2]. The limitations of such models restrict their use as a general design tool. Numerical tools thus become essential for the nonlinear analysis of structures. Some of the simplest numerical approaches include the plastic hinge method where plasticity is assumed to be concentrated at a particular point [3–5].

A practical approach to numerically investigate elastoplastic behavior is to use 1D (beam) or 2D (plate/shell) finite elements, with enriched kinematics to better describe the deformation of the 3D structure. For instance, Prokić used warping functions to describe the out-of-plane deformations in thin-walled beams [6]. Some recent developments in FEM for thin-walled beams include the Generalized Beam Theory (GBT), where cross-sectional deformation modes are computed to describe the deformed configuration. Elastoplasticity models developed using this formulation were successful in detecting localized plasticity and cross-sectional distortion in thin-walled structures without significant computational effort [7–10].

An approach to further reduce the computational cost associated with a nonlinear FE analysis is the use of global-local techniques. In general, such a procedure consists of the analysis

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of the coarsely meshed global structure, followed by the analysis over a finely discretized area of interest. The global solution is applied to the local domain as boundary conditions to drive the local analysis. Global-local techniques have frequently been used in the past decades for refined linear structural analyses, when computing power was significantly expensive [11–14]. The use of such techniques to computationally intensive nonlinear analyses is a natural progression, leading to several researchers proposing various global-local methods to solve nonlinear problems. Noor applied the global-local methodology to investigate the nonlinear and post-buckling response of composite panels [15]. Duarte et al. developed a generalized finite element method based on global-local enrichment functions and applied it to investigate problems with confined plasticity [16,17]. Gendre et al. presented a nonintrusive global-local technique for structural problems with local plasticity, using an iterative technique similar to [12], resulting in an exact structural re-analysis [18].

The objective of the current work is to predict the elastoplastic behavior of slender structures in a computationally efficient manner, by using a combination of the CUF and the global-local technique. In CUF, expansion functions are used across the beam cross-section to enrich the kinematics of the beam element, which results in 3D-like solutions at a reduced computational cost [19]. It, therefore, constitutes a suitable framework to perform nonlinear analyses. CUF has been recently extended to solve problems related to geometrical nonlinearity [20,21], and physical nonlinearity [22]. The current work extends the previous work on elastoplasticity by incorporating global-local techniques within CUF to carry out a refined analysis in the plastic zone.

The paper is organized as follows: a brief overview of CUF is given in Section 2. The concept of global-local analysis and its implementation in the CUF framework has been explained in Section 3. Some numerical results have been presented in Section 4 to validate and demonstrate the capabilities of CUF in performing nonlinear analyses. Conclusions are drawn and presented in Section 5. The Appendix A provides further details on the nonlinear implementation.

2. The Carrera Unified Formulation

The CUF is a unified framework which can be used to develop refined beam and shell/plate elements based on advanced structural theories. It uses expansion functions, F_τ, to enhance the displacements field, and hence to improve the kinematics of the FE model. For instance, the displacement field of a beam model, as shown in Fig. 1 can be described in CUF:

$$\mathbf{u} = F_\tau(x,z)\mathbf{u}_\tau(y), \quad \tau = 1,2,\ldots,M \tag{1}$$

where $F_\tau(x,z)$ is the expansion function across the cross-section, $\mathbf{u}_\tau$ is the generalized displacement vector, and M is the number of terms in the expansion function. The choice of F_τ and M are arbitrary and can be given as a user input. Two classes of expansion functions have been used for the current work, and are briefly described below.

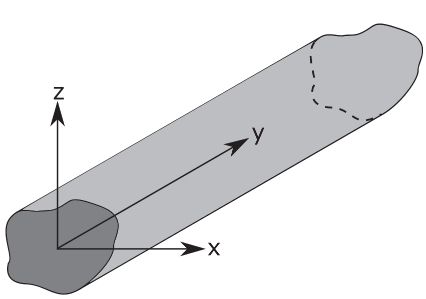

Fig. 1. An arbitrary beam element aligned with the CUF Cartesian reference system.

2.1. Taylor Expansion (TE)

In this class of expansion functions, Taylor polynomials of the kind $x^i z^j$ are used as the expansion function F_τ, over the cross-section. The order of the polynomial is denoted by N and is specified by the user. As an example, the second-order TE (N = 2, TE2), containing 18 terms, is given below,

$$\begin{aligned} u_x &= u_{x_1} + x u_{x_2} + z u_{x_3} + x^2 u_{x_4} + xz u_{x_5} + z^2 u_{x_6} \\ u_y &= u_{y_1} + x u_{y_2} + z u_{y_3} + x^2 u_{y_4} + xz u_{y_5} + z^2 u_{y_6} \\ u_z &= u_{z_1} + x u_{z_2} + z u_{z_3} + x^2 u_{z_4} + xz u_{z_5} + z^2 u_{z_6} \end{aligned} \tag{2}$$

Classical beam theories such as Euler-Bernoulli Beam Theory (EBBT) and Timoshenko Beam Theory (TBT) can be obtained as special cases of the TE. In such a formulation, the unknown degrees of freedom are the displacements and their derivatives until the N^{th} order. A detailed explanation of the TE in CUF can be found in [23].

2.2. Lagrange Expansion (LE)

In this type of expansion, the cross-section displacement field is modeled using Lagrange polynomials. In such a formulation, the unknown degrees of freedom are purely the displacements in the spatial coordinates, and no rotations are involved. As an example, the displacement field of the 9-node bi-quadratic Lagrange element (L9) is given as

$$\begin{aligned} u_x &= \sum_{i=1}^{9} F_i(x,z) \cdot u_{x_i}(y) \\ u_y &= \sum_{i=1}^{9} F_i(x,z) \cdot u_{y_i}(y) \\ u_z &= \sum_{i=1}^{9} F_i(x,z) \cdot u_{z_i}(y) \end{aligned} \tag{3}$$

where $u_{x_i}, u_{y_i}, u_{z_i}$ and F_i are the nodal translational degrees of freedom and Lagrange interpolation function of the i^{th} node, respectively. Multiple LE elements can be used to locally refined the displacement field. A detailed explanation of the LE in CUF can be found in [24].

2.3. Finite element formulation

The stress and strain tensors are represented in vector notation as follows:

$$\begin{aligned} \boldsymbol{\sigma} &= \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}\} \\ \boldsymbol{\varepsilon} &= \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yz}\} \end{aligned} \tag{4}$$

where $\boldsymbol{\varepsilon}$ is the geometrically linear strain tensor. The linear strain-displacement relation is then given by

$$\boldsymbol{\varepsilon} = \mathbf{D}\mathbf{u} \tag{5}$$

where $\mathbf{D}$ is the linear differentiation operator expressed as

$$\mathbf{D} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix}$$

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