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A wideband fast multipole accelerated singular boundary method for three-dimensional acoustic problems

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A B S T R A C T

In this paper, we present a new fast meshless method, called as wideband fast multipole singular boundary method (FMSBM), for three-dimensional acoustic problems. The wideband FMSBM applies a partial wave expansion formulation in low frequency regime and a plane wave expansion formulation in high frequency regime. The present method is efficient and accurate for a wider range of frequencies compared with the existing FMSBM approaches. In addition, the method avoids large number of element integrations in the boundary element method for resolving the complicated acoustic model, which can further reduce the computational complexity.

1. Introduction

The boundary element method (BEM) has been regarded as a powerful technique for the numerical solution of problems in computational acoustics governed by the Helmholtz equation [1–6]. The use of the BEM has several advantages over the finite element method (FEM) for problems of interest, especially in requiring only the boundary discretization and the accurate modelling of infinite domains [7–11]. The non-symmetric dense matrices appearing in the solution of the traditional BEM restrict its application to small-scale problems. To break through this bottleneck, the fast multipole method (FMM) [12,13] was introduced to improve the efficiency and reduce the memory requirement of the method [5,14–18]. However, the BEM still encounters a time-consuming issue of a large amount of numerical integrations arising from the discretization of boundary integral equations for large-scale problems [19].

During the past few years, many researchers have paid attention to the meshless methods without requirement of domain and boundary discretization. The method of fundamental solutions (MFS) [20] as a typical meshless boundary collocation approach is a competitive alternative because of its simple mathematical expression and high precision. The MFS accelerated by the FMM and the adaptive cross approximation has been applied to the simulation of large-scale problems [21,22]. Unfortunately, the traditional MFS encounters the problem of how to place the fictitious boundary outside physical domain, especially for three-dimensional complex boundary. To overcome this drawback, various numerical approaches were developed, such as regularized meshless method (RMM) [23], modified method of fundamental solutions (MMFS) [24], and singular boundary method (SBM) [25]. Among these approaches, the SBM is mathematically simple, easy-to-program, and integration-free, and has been successfully applied to solutions of various physical problems [26–34]. Thanks to these advantages, the SBM approach is less time-consuming and more applicable for complex-shaped three-dimensional domain problems than the BEM. In addition, the SBM eliminates the fictitious boundary in the MFS and becomes numerically more stable than the MFS because of better conditioned interpolation matrix.

The SBM approximates physical variables by using a linear combination of the fundamental solution of the governing equation, and its solution also generates a full interpolation matrix as in the BEM. To overcome highly computational cost of the system of equations with the full matrix, Qu et al. [35,36] developed the fast multipole singular boundary method (FMSBM) to reduce the CPU times and memory requirements of the SBM when solving three-dimensional large-scale acoustic problems. The CPU times of the FMSBM in [35] called as traditional approach is increased from $O(N)$ (N is the dimensionality of the matrix) to $O(N^2)$ when being used to solve the problems in high-frequency regime. The

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diagonal form FMSBM (DF-FMSBM) in [36] is unstable when being applied to low frequency problems.

The FMMs based on the partial wave expansion [37] and the plane wave expansion [38] fail in some way outside their preferred frequency regime. In [13], Cheng et al. firstly constructed a wideband FMM by switching the two FMM approaches depending on the level in the tree structure, which can deal with the above mentioned problems. After then, many researches [39–42] used the wideband FMM to accelerate the BEM for the solution of acoustic radiation, acoustic scattering, and acoustic shape sensitivity analysis.

In this paper, we combine the wideband FMM and the SBM to construct a wideband FMSBM for three-dimensional acoustic problems. The present approach combines advantages of the traditional FMSBM [35] and the DF-FMSBM [36], which avoids the rapidly increasing CPU times of the former for high-frequency regime and instability of the latter for low-frequency regime. The wideband FMSBM has an $O(N)$ efficiency if low-frequency computations dominate and an $O(N\log N)$ efficiency if high-frequency computations dominate. The numerical results of acoustic pressures for several numerical experiments clearly illustrate that the developed methodology is accurate and efficient. The outline of this paper is organized as follows. Section 2 provides the details of the wideband FMSBM formulations. Section 3 presents three numerical experiments, including a scattering model from a dolphin with no available analytical solution. Section 4 concludes the paper.

2. Formulations of the wideband FMSBM

2.1. The SBM formulations

In a homogeneous isotropic acoustic medium $V \in R^3$, the propagation of time-harmonic acoustic waves can be described by the Helmholtz equation

$$\nabla^2 p + k^2 p = 0, \quad p \in V \tag{1}$$

where p is the acoustic pressure, and k denotes the wave number expressed as

$$k = 2\pi f / c \tag{2}$$

in which f is the frequency of acoustic wave, and c is the wave speed. The boundary conditions are imposed as

$$p(\boldsymbol{x}) = p_1(\boldsymbol{x}), \quad \boldsymbol{x} \in S_1, \tag{3}$$

$$q(\boldsymbol{x}) = \frac{\partial p(\boldsymbol{x})}{\partial \boldsymbol{n_x}} = q_1(\boldsymbol{x}), \quad \boldsymbol{x} \in S_2, \tag{4}$$

in which $\boldsymbol{n_x}$ is the outward normal vector at point $\boldsymbol{x}$, $p_1(\boldsymbol{x}), q_1(\boldsymbol{x})$ are known functions, and the whole boundary of the domain V consists of S_1 and S_2. The acoustic pressures p for the radiation, scattering and mixed models are respectively equivalent to the following relationships

$$p = \begin{cases} p_R = p_T, & \text{for radiation,} \\ p_S = p_T - p_I, & \text{for scattering,} \\ p_R + p_S = p_T - p_I, & \text{for both,} \end{cases} \tag{5}$$

where the subscripts R, S, I, T respectively denote the radiation, scattering, incident and total waves. In addition, the acoustic pressure p for exterior acoustic wave problems has to satisfy the Sommerfeld radiation condition [43] as follows

$$\lim_{r\to\infty}\left(r\left[\frac{\partial p(r)}{\partial r} - \mathrm{i}kp(r)\right]\right) = 0, \tag{6}$$

in which $\mathrm{i} = \sqrt{-1}$, and $r = \|\boldsymbol{x}\|_2$.

For the SBM, the acoustic pressure and its normal gradient can be approximated by using a linear interpolation of the fundamental solution of the Helmholtz equation, which are respectively given as [35]

$$p(\boldsymbol{x}_i) = \sum_{\substack{j=1 \\ j\neq i}}^{m} \phi_j G(\boldsymbol{x}_i, \boldsymbol{y}_j) + \phi_i U_i, \quad i = 1, 2, \ldots, m, \tag{7}$$

$$q(\boldsymbol{x}_i) = \sum_{\substack{j=1 \\ j\neq i}}^{m} \phi_j \frac{\partial G(\boldsymbol{x}_i, \boldsymbol{y}_j)}{\partial \boldsymbol{n}_{\boldsymbol{x}_i}} + \phi_i Q_i, \quad i = 1, 2, \ldots, m, \tag{8}$$

where $\{\boldsymbol{x}_i\}_{i=1}^m$ and $\{\boldsymbol{y}_j\}_{j=1}^m$ are respectively collocation and source points, m the number of boundary points (collocation or source points), $\{\phi_j\}_{j=1}^m$ the undetermined coefficients, $G(\boldsymbol{x}_i, \boldsymbol{y}_j)$ the fundamental solution given by

$$G(\boldsymbol{x}_i, \boldsymbol{y}_j) = \frac{1}{4\pi} \frac{e^{\mathrm{i}k\|\boldsymbol{x}_i - \boldsymbol{y}_j\|_2}}{\|\boldsymbol{x}_i - \boldsymbol{y}_j\|_2}, \tag{9}$$

and U_i, Q_i the origin intensity factors expressed as

$$U_i = \frac{1}{\ell_i} \int_{S_i} G(\boldsymbol{x}_i, \boldsymbol{y}) dS_{\boldsymbol{y}}, \tag{10}$$

$$Q_i = \frac{\kappa}{2\ell_i} + \frac{1}{\ell_i} \int_{S_i} \frac{\partial G(\boldsymbol{x}_i, \boldsymbol{y})}{\partial \boldsymbol{n}_{\boldsymbol{x}_i}} dS_{\boldsymbol{y}}, \tag{11}$$

in which ℓ_i denotes the area of S_i, namely, influence domain of the origin intensity factors U_i and Q_i at source point $\boldsymbol{x}_i$, κ is set to

$$\kappa = \begin{cases} 1, & \text{for interior problems,} \\ -1, & \text{for exterior problems.} \end{cases} \tag{12}$$

With the help of Eqs. (7) and (8), we can form a system of equations as follows

$$\boldsymbol{A}\Phi = \boldsymbol{b}, \tag{13}$$

in which $\boldsymbol{A}$ is the interpolation matrix of the SBM, Φ the vector composed by the unknown coefficients, $\boldsymbol{b}$ the boundary condition of interested acoustic problems.

2.2. The wideband FMSBM formulations

In this section, we introduce the wideband FMM to accelerate the matrix-vector product in Eq. (13), and the iterative solver of generalized minimal residual method (GMRES) is then employed to solve the system of linear equations of the SBM. The fundamental solution in the wideband FMSBM is respectively expressed as into two parts: (1) a partial wave expansion formulation in low frequency regime; (2) a plane wave expansion formulation in high frequency regime. Details of the implementation of the developed method are described as follows. Fig. 1 plots the source point, collocation point, and points of multipole and local expansions in the

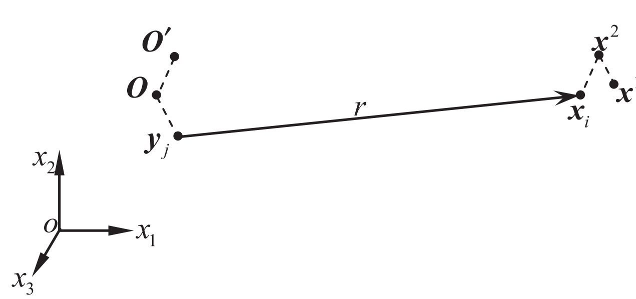

Fig. 1. Expansion points and the boundary nodes in the wideband FMSBM.