Computers and Structures xxx (2018) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Large deformation analysis of functionally graded visco-hyperelastic materials

João Paulo Pascon

Materials Engineering Department, Lorena School of Engineering, University of São Paulo, Pólo-Urbo Industrial, Gleba AI-6, s/n°, 12602-810 Lorena, SP, Brazil

ARTICLE INFO

Article history:
Received 1 March 2018
Accepted 1 June 2018
Available online xxxx

Keywords:
Large deformation analysis
Functionally graded materials
Visco-hyperelasticity
3D finite elements

ABSTRACT

In this paper, a numerical formulation for the analysis of viscoelastic functionally graded materials under finite strains is presented. The general constitutive modeling is described within the context of Lagrangian and isotropic visco-hyperelasticity. The specific models selected are the compressible neo-Hookean hyperelastic law, the Zener rheological model and the isochoric evolution law described in terms of the rate of the viscous right Cauchy-Green stretch tensor. The material coefficients may vary smooth and continuously along one direction according to the power law. The viscous update is performed via the exponential rule. The main novelty of this paper is the use of gradually variable viscoelastic coefficients in the finite strain regime.

Four numerical examples involving functionally graded materials and finite viscoelastic strains are originally analyzed to assess the formulation proposed: a bar under uniaxial extension, a block under simple shear, the Cook's membrane and an elastomeric bridge bearing. Isoparametric solid tetrahedral finite elements of linear, quadratic and cubic orders are employed. The influence of the material viscoelastic parameters on the mechanical behavior is analyzed in detail. Results confirm that mesh refinement provides more accuracy and the present model can reproduce large levels of viscoelastic strains in functionally graded materials.

1. Introduction

Viscoelastic functionally graded materials (VFGMs) have been widely used in engineering and industry. In these advanced composites, the mechanical behavior is time-dependent and the material properties vary gradually (smooth and continuously) over the volume. This gradual variation avoids the occurrence of material mismatch and stress discontinuities, which lead to delamination problems in composite laminates, for example. Functionally graded materials (FGMs) have many potential applications in high-temperature environments, e.g. nuclear reactor components, chambers of internal combustion engines, blade casing in thermal power plants and spacecraft. Such composites usually exhibit creep and relaxation viscoelastic behavior at high temperatures. A representative material having such properties is the polymer (or elastomer).

Although the damping mechanisms vary with temperature, many elastomers are viscoelastic even at room temperature, for instance. There are several works in the context of mechanical/structural analysis of VFGMs under isothermal conditions. In the work of [1], a new plate theory is developed considering the viscoelasticity of polymer foams. It is stated that, since the mechanical properties vary over the thickness direction, the foam can be approximately modeled as a FGM. As pointed out by [2], due to their impact loading resistance and low weight, polymeric foams are employed in automotives, spacecraft, submarines and airplane. To investigate the dynamic behavior, those authors develop a method for free vibration analysis of cylindrical panels composed of VFGM. In [3], a dynamic analysis of multi-span VFGM nanopipes conveying fluid is performed based on nonlocal elasticity theory, showing that the concept of FGM also has applications in nanotechnology. Moreover, since the mechanical behavior is significantly affected by the nature of spatial variation of material properties, FGMs can be tailored for specific task performances. Optimization design procedures involving VFGMs can be found, for instance, in [4] and [5]. Another fact to be highlighted is that cracks can eventually appear in FGMs under high temperatures. Examples of fracture mechanics models devoted to VFGMs are described in [6] and [7].

The usual modeling of FGMs involves the definition of a continuous function that describes the gradual variation of the material coefficients along one or more directions. The most common functionally graded (FG) models are the power law (p-FGM), the

E-mail address: jppascon@usp.br

https://doi.org/10.1016/j.compstruc.2018.06.001
0045-7949/

sigmoidal law (S-FGM) and the exponential law (E-FGM) (see, for instance, the work of [8]). Other FG models can also be adopted, such as the two- and the five-parameter exponential laws, as well as the Weibull exponential function, usually employed for carbon nanotube (CNT)-reinforced laminated composites [9–11]. For model calibration of VFGMs, the reader can be referred to the work of [12], in which the generalized Kelvin model of arbitrary order is used.

An important limitation of the abovementioned works involving VFGMs is the restriction to small strain regime. It is well known that elastomers, for example, are usually highly deformable and present nonlinear time-dependent behavior. The modeling of elastomers under finite viscoelastic strains remains a major challenge. The finite elastic response of polymers is often modeled within the context of hyperelasticity, defining a scalar strain energy function that can reproduce large strain levels and, in general, a nonlinear material model (see, for instance, the works of [13] and [14]. The inclusion of the viscoelastic behavior can be performed by setting the time-dependence of the material properties (or the damping response). Two common frameworks appear in the context of finite viscoelastic strains: Convolution Integral Model (CIM), based on convolution or hereditary integrals [15–17]; and Internal Variable Model (IVM), defined in terms of hidden (or internal) strain history variables (see, for example, the works of [18–23]). A comparison of both frameworks is done in [24], which have concluded that it is very difficult to decide which model is more appropriate for specific applications. The present formulation belongs to the second framework and can be considered as a particular case of the finite strain models proposed in [18] and [19], in which the thermal effects are considered, or the damage model suggested in [25]. The model of this paper is also restricted to isotropic materials. An example of general anisotropic visco-hyperelastic model is found in the phenomenological formulation proposed by [26].

The Finite Element Method (FEM) is employed to deal with general structural problems involving FG visco-hyperelastic materials. The element adopted is the isoparametric tetrahedral solid of any-order based on positional description, successfully employed, for instance, in [27–29]. In these works, it is demonstrated that full integration scheme together with high order polynomials is very effective to avoid locking problems even in complex structural problems, without using logarithmic strains, corotational rates and mixed formulations. The present work can be considered as an extension of the homogeneous visco-hyperelastic formulation proposed in [29] to the case in which the material has gradually variable properties. Some alternative numerical formulations have been recently proposed to solve the same problems analyzed in the present work. One example is the Consecutive-Interpolation Procedure (CIP) addressed in the works of [30–33], in which the nodal average gradients are interpolated in a second step, resulting in continuous nodal stresses without smoothing operation and without increasing the number of degrees of freedom. Another interesting approach is the Isogeometric Analysis (IGA) used, for example, in the works of [34–36]. In that formulation, the geometry is exactly described using CAD basis functions (e.g. NURBS) and fewer control points (when compared to traditional FEM), resulting in a high-order continuity and a simple mesh refinement.

According to the author's bibliographic review, there are few works in which finite visco-hyperelasticity is employed together with the concept of FGM. In the study of [37], for example, a formulation is proposed to analyze the mechanical behavior of transversely isotropic FG rubbers under finite viscoelastic strains. It is demonstrated that the proposed constitutive model agrees with uniaxial experimental data of polyurea. However, that work is theoretical and no finite element analysis is performed for general mechanical or structural problems. The purpose of the present work, motivated by the lack of studies, is to perform a finite element analysis of general 3D structural problems involving FG visco-hyperelastic materials under isothermal and quasi-static conditions. The development of this original formulation is essential mainly for elastomeric structures under finite strains and complex boundary conditions. Two novelties of the present study can be cited: the use of the FG concept to describe the gradual variation of visco-hyperelastic coefficients; and the unique finite element analysis of general structural problems involving VFGMs under finite strain levels.

This paper is organized as follows. The kinematic formulation is described in Section 2. The constitutive models, the evolution equations and the FG law adopted are provided in Section 3. The numerical solution procedure to deal with the nonlinear expressions involved is given in Section 4. The illustrative numerical examples used to validate the present methodology are described in Section 5. Finally, the main conclusions of the work are highlighted in Section 6.

2. Kinematics

The present kinematic description is the same as the one adopted in [29] and, thus, is briefly described.

The kinematics is based on the multiplicative split of the deformation gradient $\boldsymbol{F}$, similar to the Kröner-Lee decomposition employed in finite elastoplasticity:

$$\boldsymbol{F} = \frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}} = \boldsymbol{F}_{\mathrm{e}}\boldsymbol{F}_{\mathrm{v}} \tag{1}$$

where the vector fields $\boldsymbol{x}$ and $\boldsymbol{y}$ are the initial and the final (deformed) configurations, respectively; and the subscripts $()_{\mathrm{e}}$ and $()_{\mathrm{v}}$ denote, in this order, the elastic and the viscous parts. The multiplicative decomposition (1) is also used in [19,20,23,38], for instance. According to [20], the multiplicative decomposition of the deformation gradient is conceptual and is not defined experimentally.

The use of decomposition (1) leads to the concept of an intermediate configuration, described by setting $\boldsymbol{F}_{\mathrm{e}} = \boldsymbol{I} \Rightarrow \boldsymbol{F} = \boldsymbol{F}_{\mathrm{v}}$. According to [24], this configuration can be used only for quasi-static problems, which is the case of the present study. An alternative multiplicative decomposition of the gradient is found in [26], in which the reversed form is adopted: $\boldsymbol{F} = \boldsymbol{F}_{\mathrm{v}}\boldsymbol{F}_{\mathrm{e}}$. As pointed out by those authors, the intermediate configuration $\boldsymbol{F} = \boldsymbol{F}_{\mathrm{v}}$ defined in (1) is a stress-free (or relaxed) configuration only when internal static equilibrium is reached or when time scales becomes infinitely large ($t \to \infty$). However, the difference between such decompositions does not influence the present approach, since it is not necessary to determine the individual components of the gradients $\boldsymbol{F}_{\mathrm{e}}$ and $\boldsymbol{F}_{\mathrm{v}}$ (see Section 3).

The numerical approximation is based on positional description and follows the usual FEM procedure, i.e., is based on nodal positions and Lagrange shape functions. Isoparametric solid tetrahedral finite elements of any-order are employed. The degrees of freedom are the current spatial positions of the nodes, instead of nodal displacements. Further details on how to determine the deformation gradient (1) based on the nodal positions can be found, for instance, in the works of [27–29,39]. One should note that, in such references, no strain enhancement is used and, although the analysis is geometrically nonlinear, no special treatment of element distortions is carried out.

The strain measures adopted in this work are the symmetric right Cauchy-Green stretch tensor and its invariants:

$$\boldsymbol{C} = \boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} = \boldsymbol{F}_{\mathrm{v}}^{\mathrm{T}}\boldsymbol{F}_{\mathrm{e}}^{\mathrm{T}}\boldsymbol{F}_{\mathrm{e}}\boldsymbol{F}_{\mathrm{v}} = \boldsymbol{F}_{\mathrm{v}}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{e}}\boldsymbol{F}_{\mathrm{v}} \tag{2}$$

$$i_1 = \mathrm{tr}(\boldsymbol{C}) = \boldsymbol{C}_{\mathrm{e}} : \boldsymbol{C}_{\mathrm{v}} \tag{3}$$

Please cite this article in press as: Pascon JP. Large deformation analysis of functionally graded visco-hyperelastic materials. Comput Struct (2018), https://doi.org/10.1016/j.compstruc.2018.06.001