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Techniques for vibration analysis of hybrid beam and ring structures with variable thickness

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ABSTRACT

Free vibration analysis of hybrid nonlocal Euler-Bernoulli beams and non-uniform rings needs solving sixth-order differential equations with variable coefficients. In this paper, techniques are proposed to solve this problem. Discrete singular convolution beam/ring elements and weak form quadrature beam/ring elements are developed. Explicit formulas are derived and presented. The efficiency of the proposed techniques is compared to the existing advanced methods such as the differential quadrature element method and the local adaptive differential quadrature method. Selected cases of hybrid nonlocal Euler-Bernoulli beams and non-uniform rings with variable thickness are investigated. Comparisons reveal that among these methods, the proposed quadrature element method is the most efficient one. In addition, the discrete singular convolution element method with the harmonic kernel is the best efficient technique for obtaining high mode frequencies.

1. Introduction

For solving large-scale engineering problems, either strong or weak form numerical methods are frequently employed [1]. The most widely used numerical method in engineering applications is the finite element method (FEM) [2,3], which belongs to the category of the weak form method. It is shown by Tornabene et al. that the strong form methods can reach high accuracy levels with a lower number of sampling points. For the fourth order differential equations, the weak form methods with C_1 compatibility conditions can also provide as accurate results as the ones obtained by the strong form methods if the Radau of the first and second kinds and the Gauss-Lobatto-Legendre (GLL) grids are employed [1].

"Along with the ever-growing advancement of faster computing machines, the research into the development of new methods for numerical solution of problems in engineering and physical sciences also is an ongoing parallel activity. Such research interests, of course, remain motivated by needs of modern technology"[4]. Many advanced numerical techniques have been proposed thus far to overcome some inherent deficiencies existing in the conventional FEM. The differential quadrature method (DQM) and the differential quadrature element method (DQEM) [1,4–11], the local adaptive differential quadrature method (La-DQM) [12], the discrete singular convolution (DSC) [13–21] and the DSC element method [22], the integral quadrature (IQ) method [1] and the weak form quadrature element method (QEM) [23–26], are a few examples of these newly developed numerical techniques. The DQM, DQEM, La-DQM, DSC and DSC element method belong to the category of the strong form method, and the IQ method and QEM belong to the category of the weak form method.

The strong form differential quadrature element method and the weak form quadrature element method were first proposed by Striz et al. [9,23] in 1990s. Since then, the two methods have been further developed by many researchers and now projected by their proponents as potential alternatives to the conventional finite element method [1,10,11,25].

The discrete singular convolution algorithm, first proposed by Wei [13,14], is another potential alternative to the conventional finite element method. The DSC can yield not only accurate low mode frequencies, but also relatively accurate high mode frequencies [15–19]. With the help of the method of Taylor series expansion [19], the DSC element method is developed [22], which extends the DSC to analyze complex structure problems.

It is worth noting that, however, the success of the above mentioned strong and weak form element methods is only demonstrated in solving problems with the second and the fourth order differential equations or their equivalents. High-order differential equations arise in many engineering fields [12].

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The hybrid nonlocal Euler-Bernoulli beam models, possessing two or more independent small length-scale parameters, have been proposed to model the micro/nano-structures more accurately [27,28]. In the hybrid nonlocal Euler-Bernoulli beam model, the strain energy involves both local and nonlocal curvatures [27]. Thus, the free vibration behavior of hybrid nonlocal Euler-Bernoulli beams is governed by a sixth order differential equation [27–30], which is different from the nonlocal beam models governed by a fourth order differential equation [2,3,21].

Arch/ring structures are common in practice. For in-plane vibration analysis, usually three second order differential equations are simultaneously solved to get the solutions [5–8]. If only the vibration normal modes of ring structures of rectangular cross-section with a variable thickness is considered, the three second order differential equations can be combined into one sixth order differential equation [31]. In other words, normal modes of vibration of ring structures with variable thickness are governed by sixth order differential equations.

To become a well posed problem, three boundary conditions at each boundary are associated with the sixth order differential equation. Implementation of multiple boundary conditions properly is one of the keys for success in using various approximate and numerical methods. For certain simple cases, the multiple boundary conditions can be easily built into the approximate solutions [32]. For general cases, however, applying the multiple boundary conditions properly is not an easy task. In literature, either multiple degrees of freedom (DOFs) at each end point or multiple fictitious points outside the solution domain are introduced to facilitate the application of multiple boundary conditions. The number of multiple DOFs or fictitious points depends on the order of the differential equation. The element methods employ the first method to facilitate assemblages in analyzing structure problems and the La-DQM uses the second approach.

The computational efficiency is one of the major concerns in using numerical methods for solutions of complex engineering problems [1,33]. From literature review, the efficiency of the DQEM, the DSC element method and the QEM in solving sixth order differential equations have not been explored thoroughly yet, and thus is investigated in this paper. Discrete singular convolution beam/ring elements with uniformly distributed nodes and weak form quadrature beam/ring elements with any type of nodes are established for the first time. Free vibration analysis of the hybrid nonlocal Euler-Bernoulli beams and non-uniform rings is performed using the proposed element methods. The efficiency of the proposed methods in vibration analysis of hybrid nonlocal Euler-Bernoulli beams and ring structures with variable cross sections is assessed. Finally, some conclusions are drawn.

2. Theoretical formulations

Theoretical formulations of free vibration of hybrid nonlocal Euler-Bernoulli beams with constant cross sections and rings of rectangular cross-section with a variable thickness are briefly introduced first.

Let x be the longitudinal coordinate measured from the left end of the Euler-Bernoulli beam and w the transverse displacement. The beam's length, cross sectional area, and the moment of inertia are denoted by L, A and I. Young's modulus and mass density are denoted by E and ρ.

In the hybrid nonlocal Euler-Bernoulli beam model, the strain energy involves both local and nonlocal curvatures [27], thus the free vibration behavior of hybrid nonlocal Euler-Bernoulli beams is governed by a sixth order differential equation [28–30], i.e.,

$$w^{(4)} - l^2 w^{(6)} = \frac{\rho A \omega^2}{EI}\left[w - (ea)^2 w^{(2)}\right] = \lambda^2\left[w - (ea)^2 w^{(2)}\right] \qquad (1)$$

where ω is the circular frequency, symbols l as well as ea represent the independent length scale parameters, superscripts (k) represent the k-th order derivative with respect to x, and $\lambda = \omega\sqrt{\rho A/EI}$ is a non-dimensional frequency parameter.

The corresponding strain energy U and kinetic energy T of the hybrid nonlocal Euler-Bernoulli beam element are [29]

$$\frac{U}{EI} = \frac{1}{2}\int_{-L/2}^{L/2}\left\{\left[w^{(2)}\right]^2 + l^2\left[w^{(3)}\right]^2\right\}dx \qquad (2)$$

and

$$\frac{T}{EI} = \frac{\lambda^2}{2}\int_{-L/2}^{L/2}\left\{[w]^2 + (ea)^2\left[w^{(1)}\right]^2\right\}dx \qquad (3)$$

For the simply supported ring shown in Fig. 1(a) and a completely free ring shown in Fig. 2(a), the shape of the cross-section is rectangular with a constant width b and variable thickness $h(\alpha)$. Denote E, ρ, $A(\alpha)$, $I(\alpha)$ and R the Young's modulus, mass density, cross sectional area, the moment of inertia and the radius of the ring, respectively. For investigations of the normal modes of vibration of the rings, only a half ring shown in Fig. 1(b) and a quarter ring shown in Fig. 2(b) are needed to be considered [17]. As the width is constant, the unit width is considered for easy presentation.

In non-dimensional form, the differential equation for free vibration analysis of a half ring of rectangular cross-section with a variable thickness, shown in Fig. 1(b), is given by [12,31]

$$\begin{aligned}&\phi w^{(6)} + 3\phi^{(1)} w^{(5)} + \left[2\pi^2\phi + 3\phi^{(2)}\right]w^{(4)} + \left[4\pi^2\phi^{(1)} + \phi^{(3)}\right]w^{(3)}\\&+\left[\pi^4\phi + 3\pi^2\phi^{(2)}\right]w^{(2)} + \left[\pi^4\phi^{(1)} + \pi^2\phi^{(3)}\right]w^{(1)}\\&= \lambda^2\pi^4\left(Pw^{(2)} + P^{(1)}w^{(1)} - \pi^2 Pw\right)\end{aligned} \qquad (4)$$

In Eq. (4), $P(x) = 1 - 4(r-1)(x^2 - x)$, $\phi = P(x)^3$, $\lambda = \omega R^2\sqrt{\rho A(0)/EI(0)}$, where ω is the circular frequency, $x = \alpha/\pi$ and $r = h(\pi/2)/h(0)$ reflecting the variation degree of the thickness of the ring structure.

The strain energy U and kinetic energy T corresponding to Eq. (4) are given by [31]

$$T = \frac{\lambda^2}{2}\int_0^1 P(x)\left\{\pi^2(w)^2 + \left[w^{(1)}\right]^2\right\}dx \qquad (5)$$

and

$$U = \frac{1}{2}\int_0^1 \phi(x)\left[w^{(1)} + \frac{1}{\pi^2}w^{(3)}\right]^2 dx \qquad (6)$$

In the non-dimensional form, the differential equation for free vibration analysis of a quarter ring of rectangular cross-section with a variable thickness, shown in Fig. 2(b), is given by [12,31]

$$\begin{aligned}&16\phi w^{(6)} + 48\phi^{(1)} w^{(5)} + \left[8\pi^2\phi + 48\phi^{(2)}\right]w^{(4)}\\&+16\left[\pi^2\phi^{(1)} + \phi^{(3)}\right]w^{(3)} + \left[\pi^4\phi + 12\pi^2\phi^{(2)}\right]w^{(2)}\\&+\left[\pi^4\phi^{(1)} + 4\pi^2\phi^{(3)}\right]w^{(1)}\\&= \lambda^2\pi^4\left(Pw^{(2)} + P^{(1)}w^{(1)} - \pi^2 Pw\right)\end{aligned} \qquad (7)$$

In Eq. (7), $P(x) = 1 - (r-1)(x^2 - 2x)$, $\phi = P(x)^3$, $\lambda = \omega R^2\sqrt{\rho A(0)/EI(0)}$, where ω is the circular frequency, $x = 2\alpha/\pi$ and $r = h(\pi/2)/h(0)$ which reflects the variation degree of the thickness of the ring structure. Note that Eq. (7) is different from Eq. (4).

The strain energy U and kinetic energy T corresponding to Eq. (7) are given by [31]

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