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Evolutionary topology optimization of continuum structures under uncertainty using sensitivity analysis and smooth boundary representation

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ABSTRACT

This paper presents an evolutionary approach for the Robust Topology Optimization (RTO) of continuum structures under loading and material uncertainties. The method is based on an optimality criterion obtained from the stochastic linear elasticity problem in its weak form. The smooth structural topology is determined implicitly by an iso-value of the optimality criterion field. This iso-value is updated using an iterative approach to reach the solution of the RTO problem. The proposal permits to model the uncertainty using random variables with different probability distributions as well as random fields. The computational burden, due to the high dimension of the random field approximation, is efficiently addressed using anisotropic sparse grid stochastic collocation methods. The numerical results show the ability of the proposal to provide smooth and clearly defined structural boundaries. Such results also show that the method provides structural designs satisfying a trade-off between conflicting objectives in the RTO problem.

1. Introduction

Topology optimization aims at finding the optimal layout of material within a design domain for a given set of boundary conditions such that the resulting material distribution meets a set of performance targets [1]. Contrary to other disciplines within structural optimization such as size and shape optimization, in topology optimization the material distribution is obtained without assuming any prior structural configuration. This provides a powerful tool to find the best conceptual design that fulfills the requirements at the early stages of the structural design [2]. Such a method has been successfully applied to a wide range of problems, from nanophotonics design [3] to aircraft and aerospace structural design [4,5], which validates it as an effective tool for least-weight and performance design.

Topology optimization methods can be broadly classified, following [6], into density-based methods [7,8], level set methods [9,10], phase field methods [11,12], topological derivative methods [13,14] and evolutionary approaches [15]. The variants of Evolutionary Structural Optimization (ESO) method [16] are some of the approaches included in the last category, such as the Bi-directional Evolutionary Structural Optimization (BESO) method [17] and the Evolutionary Topology Optimization (ETO) method using isolines [18–20] and smoothing boundary representation [21]. These optimization methods are based on heuristic rules including from simple hard-kill strategies (elements with lowest strain energy density are removed) to bidirectional schemes (elements can be reintroduced if considered rewarding). Apart from intuition, such methods can use standard adjoint gradient analysis and filtering techniques to stabilize algorithms and results [22,23].

ETO methods have shown their ability for providing structurally sound and aesthetically pleasing designs [24], which commonly mimic nature's own evolutionary optimization process. Such methods normally assume deterministic conditions to integrate function and form in a synergistic way [25], which obviates the different sources of uncertainty that may affect not only the safety and reliability of structures but also their performance. These sources of uncertainty include epistemic uncertainties, typically due to limited data and knowledge, and aleatory uncertainties, which are the natural randomness in a process, including manufacturing imperfections, unknown loading conditions, variations of the material properties, etc. The introduction of uncertainty to model realistic conditions in the design process has shown to be a key issue for solving real-world engineering problems in several fields, such as civil [26], automotive [27] and mechanical [28] engineering, to name but a few. This fact, together with the

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development of probabilistic uncertainty propagation methods, has fostered the interest for considering uncertainty within the topology optimization problems, giving rise to the formulation of several approaches embraced under the term of Topology Optimization Under Uncertainty (TOUU) methods.

TOUU methods can be broadly classified, according to the representation and treatment of uncertainties, into non-probabilistic and probabilistic approaches. Non-probabilistic approaches [29] do not require the statistical information about the uncertainty of the phenomenon but a qualitative notion about its magnitude. The worst-case approach [30,31], taking the form of a min–max optimization problem, and fuzzy techniques [32], making use of fuzzy set theory, are some of the methods included in this category. The main drawback of these approaches is that they are often too conservative, due to overestimation of uncertainty, and may lead to optimal designs with poor structural performance. Conversely, probabilistic methods make use of a probabilistic characterization of the uncertainty of the phenomenon. Several formulations have been proposed in this context, which differs from each other in the design of the objective function as well as in the way the uncertainty is incorporated in the formulation.

Reliability-Based Topology Optimization (RBTO) aims at determining the best design solution with respect to prescribed criteria, e.g. stiffness, weight and construction costs, while explicitly considering the unavoidable effects of uncertainty. This is done by defining the constraints in terms of the probability of constraint violation (probability of failure) [33]. Risk-Averse Topology Optimization (RATO) [34,35] aims at minimizing a risk cost function that quantifies the expected loss related to the damages, such as excess probability. That is, whereas RBTO provides optimal designs in terms of deterministic prescribed criteria with enough reliability level, RATO provides the best design from the point of view of risk-aversion [36]. Contrary to RBTO and RATO formulations, Robust Topology Optimization (RTO) incorporates statistical moments of the compliance to the objective function. The aim is to obtain optimal designs which are less sensitive to variations in the input data. Several developments based on RTO formulation have been developed to handle uncertainty in loading [37], material [38], stiffness [39], geometry [40], boundary [41], and loading and material [42,43]. For the specific case of uncertainty in loading, [44] [44] and then [45] [45] showed that the RTO problem of minimizing the expected compliance is analogous to a multiload-like problem associated with a particular finite set of loading scenarios, which depend on the mean and the variance of the perturbations [46]. Nevertheless, a very common practice is to use the weighted sum of the first two statistical moments of compliance as the objective function of RTO formulation [43,38,37]. This is the formulation adopted in this work, where the expected value and the standard deviation of the compliance are considered as a measure of structural robustness.

Despite the fact that ETO methods have been successfully applied to the design of many complex industrial deterministic problems, they have not been used to the same extent to address TOUU problems. Kim et al. [47] addressed the RBTO problem using the ESO method and first-order reliability approach, as approximate probability integration method, to solve problems with uncertainty in loading and material. Eom et al. [48] made use of an improved hard-kill BESO method using a response surface to compute the reliability index for addressing RBTO problems with uncertainty in loading and material. The BESO method using a performance measure, with probabilistic constraints formulated in terms of the reliability index, was used by Cho et al. [49] to address RBTO multi-objective problems including uncertainty in static stiffness of bending, torsion, and natural frequency. The linear elasticity hypothesis was exploited by Kanakasabai and Dhingra [50] using superposition to efficiently handle reliability constraints in RBTO problems with uncertainty in loading using the BESO method. Recently, topology optimization of continuum structures under probabilistic and fuzzy loads is addressed by Liu et al. [51] using BESO method; in particular, the uncertainty of input data is described using a cloud model that permits to transform the uncertain topology optimization problem into a deterministic one with multiple load cases.

In this work, an ETO method driven by an optimality criterion is proposed for addressing the TOUU problem. This proposal includes some of the ingredients of the iso-XFEM method [19]; in particular, the use of implicit boundary representation by iso-contours, to control the shape and topology variations during the optimization process, and the extended finite element method (XFEM), to improve the accuracy of finite element solutions on the boundary of the design. The optimality criterion for the RTO problem is derived from the stochastic linear elasticity formulation in its weak form using a continuous adjoint method without being limited by the discretization method used for the physical and the stochastic domains. The ETO method uses an iterative approach to gradually add and/or remove material based on the iso-contours of the optimality criterion. The proposal permits to handle loading and material uncertainties modeled by different probability distributions and random field. To address the increment of dimensionality induced by the random field, an anisotropic sparse grid stochastic collocation method is used for the efficient computation of the multidimensional integrals over the random domain. Compared to density-based and level-set methods addressing TOUU problems, the proposal requires neither an initialization of the boundary nor any regularization parameter, and it provides smooth and clearly defined boundaries. Another important advantage of the proposal is that it provides optimal solutions for different volume fractions during the optimization process, which enables to efficiently find a trade-off between performance and robustness for different volume fractions during the topology optimization.

The remainder of the paper is organized as follows. The basis and theoretical background of TOUU problem and the RTO formulation are briefly reviewed in Section 2. Section 3 presents the adaptive sparse-grid stochastic collocation method used for uncertainty propagation and the efficient computation of the multidimensional integrals over the random domain required by the RTO formulation. The optimality criterion, used by the proposal to reach an optimal solution, is derived for the RTO problem in Section 4. Section 5 presents the proposed ETO algorithm driven by an optimality criterion to address the RTO problem. Section 6 is devoted to the numerical experiments used for validating the proposed method. Finally, Section 7 presents the conclusion of the proposed ETO method for RTO problems.

2. Topology optimization under uncertainty (TOUU)

The mathematical basis and fundamentals of TOUU problems and the specific formulation of RTO, addressed in this work, are presented below.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space, and let $D \subset \mathbb{R}^d$ ($d = 2$ or $d = 3$) be a bounded Lipschitz domain whose boundary is decomposed into three disjoint parts $\partial D = \Gamma_D \cup \Gamma_N \cup \Gamma_0$. Consider the linearized elasticity system under random input data

$$\begin{cases} -\nabla \cdot \sigma(u(x,\omega)) = b(x,\omega) & \text{in } D \times \Omega \\ u(x,\omega) = \bar{u} & \text{in } \Gamma_D \times \Omega \\ \sigma(u(x,\omega)) \cdot n = \bar{t}(x,\omega) & \text{in } \Gamma_N \times \Omega \\ \sigma(u(x,\omega)) \cdot n = 0 & \text{in } \Gamma_0 \times \Omega \end{cases}, \tag{1}$$

where x is the spatial variable, $\omega \in \Omega$ are the random events, σ is the Cauchy stress tensor, b and $\bar{t}$ are the body and surface forces, $\bar{u}$ is the prescribed displacement field, and n is the unit outward