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Solving generalized eigenvalue problems for large scale fluid-structure computational models with mid-power computers

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ABSTRACT

This article proposes a method for solving generalized eigenvalue problems on medium-power computers with a moderate memory in the particular context of studying fluid-structure systems with sloshing and capillarity. This research was performed following many RAM problems encountered when computing the modal characterization of the system studied. The methodology proposed is one solution to reduce RAM and time required for the computation, by using methods such as double projection or subspace iterations.

1. Introduction

The algorithms for solving eigenvalue problems (including generalized eigenvalue problems for which one matrix is positive definite) have received a very great attention this last 40 years from a mathematical point of view (see for instance, [1–9]), for algorithms adapted to parallel computation (see for instance, [10–18]), and also for massively parallel computers (see for instance, [19–23]). The majority of the efficient algorithms have been implemented in a mathematical library for computers, parallel computers, and massively parallel computers (see for instance, [24–26]).

This paper is devoted to the computation of very populated sparse matrices involved in generalized eigenvalue problems that have to be solved in the framework of fluid-structure problems. Concerning the algorithms for solving these generalized eigenvalue problems for which one of the two matrices is a positive-definite matrix, the mathematical libraries cited before could, *a priori*, be used (these algorithms are really efficient and are adapted to large scale models using parallel and massively parallel computers). Although these algorithms are efficient on *mid-power computers* that we define as workstations with, for instance, 264 GB to 1 TB for the RAM and 12–72 cores for the processors, we have encountered huge difficulties due to the limitation of RAM and also to CPU-time consumption.

The framework of the developments proposed is the one relative to the computation of reduced-order bases (ROB) in order to construct a reduced-order model (ROM) of a fluid-structure computational model that corresponds to an elastic structure coupled with an internal acoustic liquid with a free surface for which there are sloshing phenomena and surface tension effects. This ROM is not constructed using a global ROB associated with the full coupled problem, but is constructed using the elastic modes of the structure with the added-mass effects, the acoustic modes of the liquid, and the sloshing/capillarity modes. The interest of such a formulation (see [27–29]) is to be able to select the modes that contribute to the responses in the frequency band of analysis and also to be able to implement the nonparametric probabilistic approach of model uncertainties in each part of the coupled system for which the level of uncertainties differs from a part to another one. It should be noted that this formulation differs from the vibroacoustics problems (without sloshing and surface tension effects) for which a ROM is constructed using a global ROB (see for instance [30]). The difficulties encountered in the computation depends on the type of modes that have to be computed. Concerning the computation of the elastic structural modes, the mass matrix of

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Nomenclature

h_K	fluid layer height
$\mathbf{h}$	free surface elevation vector
n_F	number of acoustic dofs
n_H	number of sloshing dofs
n_S	number of structural dofs
$\mathbf{p}$	acoustic pressure vector
$\mathbf{u}$	structural displacements vector
$C_{p\eta}$	fluid-sloshing coupling matrix
$\mathcal{C}_{p\eta}$	projection of $C_{p\eta}$
$C^a_{p\eta}$	approximation of $C_{p\eta}$
C_{pu}	fluid-structure coupling matrix
$\mathcal{C}_{pu}$	projection of C_{pu}
$\mathbb{I}_n$	$(n \times n)$ identity matrix
K_F	acoustic stiffness matrix
K^a_F	approximation of K_F
K_{gc}	free surface stiffness matrix
K_S	structural stiffness matrix
M_A	added mass matrix
$\mathcal{M}_A$	projection of M_A
M_F	acoustic fluid mass matrix
M_{gc}	free surface mass matrix
M^a_{gc}	approximation of M_{gc}
$\mathcal{M}_{gc}$	projection of M_{gc}
M_S	structural mass matrix
N_d	dimension of initial sloshing subspace
N_F	number of acoustic eigenvalues
N_H	number of sloshing eigenvalues
N_p	dimension of subspace projection basis
N_q	dimension of initial elastic subspace
N_S	number of elastic eigenvalues
$\mathcal{R}_F$	subspace of $\mathbb{R}^{n_F}$ with $\mathbf{p} = 0$ on free surface
$\lambda_i^{F,\text{ref}}$	$i^{th}-$ eigenvalue in Λ_F^{ref}
Λ_F	acoustic eigenvalues matrix
Λ_F^{ref}	reference acoustic eigenvalues matrix
$\lambda_i^{H,\text{ref}}$	i^{th} – eigenvalue in Λ_H^{ref}
Λ_H	sloshing eigenvalues matrix
Λ_H^{ref}	reference sloshing eigenvalues matrix
$\lambda_i^{S,\text{ref}}$	i^{th} – eigenvalue in Λ_S^{ref}
Λ_S	elastic eigenvalues matrix
Λ_S^{ref}	reference elastic eigenvalues matrix
Φ_F	acoustic eigenvectors matrix
Φ_{FH}	sloshing eigenvectors block matrix
Φ^a_{FH}	approximation of Φ_{FH}
Φ_F^{ref}	reference acoustic eigenvectors matrix
Φ_H	sloshing eigenvectors block matrix
Ψ_H	sloshing eigenvectors matrix
Ψ_H^{ref}	reference sloshing eigenvectors basis
Φ_S	elastic eigenvectors matrix
Φ_S^{ref}	reference elastic eigenvectors matrix
Φ_H^{ref}	reference sloshing eigenvectors block matrix

the generalized eigenvalue problem is made up of the sparse mass matrix of the structure in which is added the added-mass matrix of the internal liquid (the added-mass matrix is a full matrix with respect to the fluid-structure coupling dofs). Due to a RAM consumption problem, the computation of the added-mass matrix cannot be done as soon as the acoustic-stiffness matrix of the internal liquid is very populated. In addition, assuming that the added-mass matrix has been computed, if the stiffness matrix of the structure is also very populated, another difficulty arises for solving the generalized eigenvalue problem inducing the same type of RAM consumption. The difficulties are exactly of the same nature for the computation of the sloshing/capillarity modes. Concerning the computation of the acoustic modes of the internal liquid, the difficulties are due to the generalized eigenvalue problem that involves two very populated sparse matrices, the acoustic mass and the acoustic stiffness matrices. These difficulties are detailed in Section 5 for which the fluid-structure computational model has 2×10^6 dofs and requires, among others, to solve a linear equation for a positive-definite matrix that has 1.2×10^8 nonzeros entries requiring about 10^9 bytes.

Confronted with this situation, we have thus revisited the formulations in order to be able to solve the three generalized eigenvalue problems on a mid-power computer. The authors think that the substantial efforts, which have been performed, could be of interest for the community. It should be noted that the formulations/algorithms proposed allow for computing a large scale fluid-structure computational model on mid-power computers but certainly, would allow for computing very large scale fluid-structure computational models on high-power computers.

The computational model of the considered fluid-structure system is constructed using the finite element method, assuming the structure is linear elastic and the internal acoustic liquid is dissipative. The free surface of the liquid is submitted to an acceleration field independent of time such as the gravitation field, inducing sloshing phenomena. The surface tension effects are taken into account.

In the particular context of this fluid-structure interaction problem for which sloshing and surface tension effects are taken into account, many research have been performed (see for instance, [27,31–33]). In this paper, the formulation used is the one presented in [28,29] for which the adapted reduced-order model (ROM) has been evoked and is more detailed hereinafter. The construction of the ROM requires a modal characterization of the different parts of the fluid-structure system. It consists in projecting the computational model using three ROB's that are computed by solving three generalized eigenvalue problems. The modal characterization of the structure is obtained by computing the elastic eigenmodes of the structure taking into account the influence of the internal acoustic liquid in order to assure a fast convergence with respect to the number of elastic modes retained in the ROM. The modal characterization of the internal acoustic liquid is obtained by computing the acoustic modes with a free surface on which the pressure is zero. Finally, the modal characterization of the free surface in presence of surface tensions is obtained by computing the sloshing modes that involve the internal acoustic liquid. The finite element meshes of the fluid-structure system that will be considered in Section 5 have a large number of dofs and a high connectivity, inducing very populated sparse matrices and consequently, leading us to an impossibility to construct the matrices and to compute the generalized eigenvalue problems on mid-power computers using the most adapted algorithms available in the mathematical libraries such as those proposed in Matlab.

Concerning the choice of the formulation, two possibilities can be envisaged. For computing the structural elastic modes with the added-mass effects or for computing the sloshing modes with capillarity effects, a first formulation could be based on the use of iterative algorithm for solving linear matrix equation (relative to all the physical dofs) for a very populated matrix and for a large number of right-hand side members. A second formulation would avoid to solve such a linear systems of equations in high dimension by using a double projection method, also known as the Rayleigh-Ritz method in the framework of eigenvalue problems. An analysis

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