Computers and Structures xxx (2018) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

An explicit-implicit projection approach for solving saddle-point systems arising from parameter identification problems

Naoufal Nifa [a,b,*], Denis Aubry [a], Mathieu Corus [b]

[a] MSSMat, CNRS, CentraleSupélec, Université Paris-Saclay, 8-10 rue Joliot-Curie, 91190 Gif-sur-Yvette, France
[b] IMSIA, UMR EDF-CNRS-CEA-ENSTA ParisTech 9219, 7 boulevard Gaspard Monge, 91120 Palaiseau, France

ARTICLE INFO

Article history:
Received 10 February 2018
Accepted 8 May 2018
Available online xxxx

Keywords:
Constrained optimization
Saddle point systems
Block preconditioners
Inverse problems

ABSTRACT

This paper proposes a new approach to solve sparse linear saddle-point systems arising in large scale parameter estimation approach using energy functionals. The constraints of those systems involve kinematic constraints and sensors ones. The approach is based on a double projection of the generated saddle point system onto the nullspace of the constraints. The first projection onto the kinematic constraints is proposed as an explicit process through the computation of a sparse null basis. Then, we detail the application of a constraint preconditioner within a Krylov subspace solver, as an implicit second projection of the system onto the nullspace of the sensors constraints. We further present and compare approximations of the constraint preconditioner. The approach is implemented in a parallel distributed environment. Significant gains in computational cost and memory are illustrated on several industrial applications in comparison to direct solvers.

1. Introduction

It is essential for industry to understand the mechanical behavior of materials to ensure their structural performance and optimize their availability. Some structures may be exposed to high levels of vibration which requires a deep understanding of the physical phenomena involved and the realization of numerical models to assess the corrective solutions.

In order to diagnose the origin of the problem, test campaign is first and foremost performed on structures. A numerical model is then built to reproduce the nominal behavior and evaluate proposed solutions. Its adequation is quantified during the stages of verification and validation and experimental information is combined with numerical simulations to fulfill the *a priori* knowledge of structural behavior to propose industrial solutions. These steps above describe in fact an inverse problem of model properties identification. It generally ends up being formulated as an optimization problem, namely seeking the minimum of a cost function that quantifies in a certain metric the difference between a model prediction and the available data [2]. Among the different existing approaches for building suitable cost function, one is based on energy functionals. This approach has shown its efficiency and has appeared to be an appropriate indicator of the quality of a model with respect to measured data. Actually, it is able to locate erroneously modeled regions in space [6,20], robust even in presence of noisy data and provides good convexity properties of cost functions [19].

For most of the industrial and application cases, and in the particular scope of interest of this work, the study of structural dynamic behavior is performed by means of Finite Element (FE) method. Adopting the above approach in a FE framework leads to a large and sparse linear system which, as recommended by industrial guidelines of this work, is formulated symmetrically. This symmetric formulation generates a sparse and large linear system of equations equivalent to a saddle-point or Karush-Kuhn-Tucker (KKT) system. It arises similarly in many applications of scientific computing, like constrained optimization and incompressible fluid flow. A review of the most known resolution techniques is found in [7].

Many reasons explain why energy-based functional approach is less often implemented. Actually, the special structure of the resulting linear system mentioned above is a difficult challenge especially for mechanical softwares which are more developed for FE-like matrices. These softwares mainly using direct methods and factorization, are often used for their robustness and the moderate storage requirement of mechanical problems. However, when applied on these symmetric indefinite matrices, they are

* Corresponding author at: MSSMat, CNRS, CentraleSupélec, Université Paris-Saclay, 8-10 rue Joliot-Curie, 91190 Gif-sur-Yvette, France.
E-mail addresses: naoufal.nifa@centralesupelec.fr (N. Nifa), denis.aubry@centralesupelec.fr (D. Aubry), mathieu.corus@edf.fr (M. Corus).

https://doi.org/10.1016/j.compstruc.2018.05.004
0045-7949/

more than often inefficient due to a significant growth factor and a high fill-in. The repeated use of the energy-based functional approach for model updating [16] or robust expansion applications, leads then to a huge computational cost. Many techniques based on model reduction have accelerated calculations by downgrading the error localization properties [9,13]. Here, wishing to keep these localization properties, the problem is solved without further approximation.

Also, the implementation of energy-based functional approach within recent work [22] shows that direct solvers used in mechanical softwares fail to solve efficiently the inverse problem associated to an industrial structure model with more than 10^6 dofs and few hundreds of measurement points and provide a huge computation cost for a single calculation.

Many existing resolution methods are used to solve the saddle-point problems, a review of the most known resolution techniques is found in [7]. While most used solvers in mechanical studies are coupled (or global) ones which enables to solve the whole system at once and then to compute the unknowns simultaneously, efficient saddle point solvers are generally block-wise designed using iterative approaches as will be shown in (2.3).

This paper proposes another class of solvers, able to handle the studied sequence of saddle point linear systems with negligible memory cost. It subsequently studies the adequacy of a projection onto the nullspace of the constraints to address problems of industrial relevance. It also contributes to the research area related to algebraic block preconditioning. The proposed approach is made possible by making a distinction in the existing constraints. While kinematic constraints are fixed linear and affine conditions, the constraints related to sensors degrees of freedom are varying along the sequence of saddle point linear systems. This distinction has dictated the type of projection of each kind of constraints.

The article is organized as follows. The inverse problem of energy functional approach is introduced in Section 2. The structure of the resulting system is also described, and existing iterative saddle point solvers are presented. The research results of Section 3 are the main contribution of the paper where a new algorithm is introduced. Section 4 is devoted to numerical results based on academic and industrial applications. Finally, concluding remarks end the paper in Section 5.

2. Background

A common approach for estimating model parameters is to solve a constrained optimization problem by minimizing the distance between measured and computed responses. Energy functional approach is based on a cost functional that measures model error, in terms of a norm of energy.

2.1. The constrained optimization problem

Let us consider a structure and its FE model with n degrees of freedom (dofs), where $[M] \in \mathbb{R}^{n\times n}$ and $[K] \in \mathbb{R}^{n\times n}$ are the so-called mass and stiffness matrices respectively. We know that each couple of eigenvalue and eigenvector (ω, φ) of the finite element numerical model satisfies

$$([K] - \omega^2[M])\{\varphi\} = 0, \{\varphi\} \neq 0. \tag{2.1}$$

Due to modeling assumptions, simplifications, misconceptions and possible model errors, numerical eigencouples may not correspond to the real dynamic behavior of the structure. We use a set of unknown model parameters θ that parametrize the mass matrix $[M] = [M(\theta)] \in \mathbb{R}^{n\times n}$ and the stiffness matrix $[K] = [K(\theta)] \in \mathbb{R}^{n\times n}$, and consequently couples of eigenvalue and eigenvector $(\omega_\theta, \varphi_\theta)$, in order to modelize these uncertainties and mis-knowledge.

The identification problem aims to find this set of parameters θ such that each couple of numerical eigenvalue and eigenvector $(\omega_\theta, \varphi_\theta)$ is close to the correspondent experimental one $(\omega_{exp}, \phi_{exp})$ where ϕ_{exp} is only defined on $s \ll n$ sensors.

The energy functional is constructed using two fields:

- Let $\{\varphi\}$ be interpreted as the best estimation of the eigenmode φ_θ, minimizing the distance with the measured eigenmode ϕ_{exp} at the pulsation ω_{exp}.
- Let $\{\psi\}$ be an error field that expresses the error in stiffness in the model which facilitates identifying the best set of parameters θ that enables a satisfactory reproduction of the measurements through successive iterations. It satisfies:

$$[K(\theta)]\{\psi\} = \Big([K(\theta)] - \omega^2_{exp}[M(\theta)]\Big)\{\varphi\}. \tag{2.2}$$

The energy functional consists of the elastic potential energy of the error term ψ and augmented by the distance between measured and computed eigenmodes:

$$\begin{aligned} e_\omega(\{\varphi\},\{\psi\},\{\theta\}) = &\frac{1}{2}\{\psi\}^T[K(\theta)]\{\psi\} \\ &+ \frac{r}{2(1-r)}(\Pi\{\varphi\} - \{\phi_{exp}\})^T[K_r] \\ &\times (\Pi\{\varphi\} - \{\phi_{exp}\}) \end{aligned} \tag{2.3}$$

where $r \in [0,1]$ is a weighting scalar, Π is a projection operator from the space of the numerical finite element model to the observation space and $K_r \in \mathbb{R}^{s\times s}$ is a symmetric positive definite scaling matrix. Although the choice of K_r is not *a priori* defined, it is usually chosen to be dimensionally consistent with the induced energy norm and can be obtained, for instance, by using the Guyan reduction on the observation space. More details about K_r are available in [2].

In addition to the constraint (2.8), there are kinematic linear constraints which are described as follows

$$[C]\{\varphi\} = 0, \quad [C]\{\psi\} = 0, \tag{2.4}$$

where $[C] \in \mathbb{R}^{m\times n}$ represents m linear relations coming from the kinematic boundary conditions and modeling constraints. It is supposed to be of full row rank m. If it is not the case, we find either that the problem is inconsistent or that some of the constraints are redundant and can be deleted without affecting the solution of the problem. Moreover, the matrix $[K]$ is supposed to be positive definite on $\ker([C])$, which ensures that the constraints lock the rigid body motions of the structure.

The experimental measurements here play the role of Tikhonov regularization parameters [32,10], which facilitates the reconstruction of unobserved fields. The parameter r makes it possible to control the importance of regularization in the cost function. It represents the confidence that we have in the identified eigenvectors. Actually, the more the coefficient r tends to 1, the more the motion of identified solution degrees of freedom corresponds to the motion of the experimental degrees of freedom. On the contrary, the more r tends to 0, the more the motion of the identified solution degrees of freedom tends towards the motion of the numerical model degrees of freedom. In the many publications dealing with energy-based approaches for identification problems, few provide a real justification for the value of r, and choosing its suitable value is not trivial. Nevertheless, the value of $r = 0.5$ makes the cost function robust with respect to the noise as

Please cite this article in press as: Nifa N et al. An explicit-implicit projection approach for solving saddle-point systems arising from parameter identification problems. Comput Struct (2018), https://doi.org/10.1016/j.compstruc.2018.05.004