Computers and Structures 203 (2018) 18–33

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Reduced-order modelling using nonlinear modes and triple nonlinear modal synthesis

Colas Joannin [a,b,*], Fabrice Thouverez [b], Benjamin Chouvion [b]

[a] Safran Helicopter Engines, 64511 Bordes, France
[b] École Centrale de Lyon, LTDS, CNRS UMR 5513, 69130 Écully, France

ARTICLE INFO

Article history:
Received 17 September 2017
Accepted 9 May 2018

Keywords:
Reduced-order model
Nonlinear mode
Characteristic constraint mode
Interface mode
Component mode synthesis
Mistuned bladed disk

ABSTRACT

This paper introduces a novel reduced-order modelling technique well-suited to the study of nonlinear vibrations in large finite element models. The method combines the concepts of nonlinear complex modes and characteristic constraint modes – or interface modes – to build a fully algebraic reduced-order model easily solved by standard iterative solvers. The performance of the method is appraised on a nonlinear finite element model of bladed disk in the presence of structural mistuning, highlighting the potential of this new technique to deal with current industrial problematics.

1. Introduction

Despite the growing computing capabilities provided by recent advances in computer science and numerical analysis, engineers are still facing some limitations when it comes to modelling large-scale industrial structures, essentially due to the size of the mathematical models yielded by standard discretisation techniques. These limitations make reduced-order modelling a key topic of research for the structural dynamics community, and the literature is nowadays rich of many contributions. Classic component mode synthesis (CMS) methods [1–5] are quite famous in mechanical engineering, mainly appreciated for their flexibility, and are nowadays implemented in most commercial finite element (FE) softwares. Various improvements to these techniques have been suggested [6], but some difficulties remain, especially when the system is subjected to strong nonlinearities. Nonlinear phenomena are yet ubiquitous in classical physics, and even if a linear analysis can often provide a good insight into the behaviour of the system, taking into accounts nonlinearities in the simulations may sometimes prove necessary.

The present work aims at contributing to the endeavours of the scientific community in providing the industry with novel and efficient reduced-order modelling techniques well suited to the study of nonlinear systems. This paper introduces a new method relying on a substructuring approach to benefit from the flexibility of standard CMS techniques. The key feature of this method is the use of so-called nonlinear modes to capture the nonlinearities in the reduction basis. First initiated by Rosenberg [7–9] in the 1960s, the concept of nonlinear mode, which can be seen as an extension of linear modes to nonlinear systems, has been broadened by the contributions of many authors since then [10–20]. In addition to the literature that provides today researchers and engineers with numerous analytical and numerical techniques to calculate nonlinear modes, thoroughly reviewed by Renson et al. in [21], the improving hardware and software capabilities provided by modern scientific computing make it possible nowadays to apply it to industrial structures.

The reduced order modeling technique derived in this paper can be split into three successive reduction steps yielding a parametric reduced-order model (ROM) retaining only a few generalised coordinates. To tackle the computation of steady-state vibrations, this ROM is then turned into a set of simultaneous algebraic equations by assuming a multiharmonic solution, in a similar fashion to the harmonic balance method (HBM). Frequency-based techniques such as the HBM have proved very efficient in the past to study the forced response of nonlinear systems, especially when dealing with localised nonlinearities since they allow an exact condensation of the problem on the nonlinear degrees of freedom (DOF) [22,23]. However, these methods are still struggling to handle large finite element models comprising thousands of nonlinear

* Corresponding author at: École Centrale de Lyon, LTDS, CNRS UMR 5513, 69130 Écully, France.
E-mail address: colas.joannin@doctorant.ec-lyon.fr (C. Joannin).

https://doi.org/10.1016/j.compstruc.2018.05.005
0045-7949/

variables. The method derived hereafter overcomes this difficulty by retaining nonlinear eigenvectors in the reduction basis, allowing to treat the nonlinear DOFs as slave coordinates of the reduction. Moreover, most of the time frequency-based appoaches require a systematic switch from frequency domain to time domain in order to evaluate the nonlinear forces through a so-called alternating frequency-time (AFT) scheme [24,25], which can be computationally intensive, though providing great flexibility as to the nature of the nonlinearities that can be tackled. To circumvent this numerical cost, it is proposed here to approximate the nonlinearities using the nonlinear eigenvalues of the nonlinear modes in a procedure later referred to as spectral substitution. The approach derived in this paper shares some similarities with the nonlinear modal synthesis procedure proposed by Krack et al. [24] and used to compute the nonlinear forced response of a bladed-disk. However, the method used in [24] assumed a perfectly cyclic structure to compute the nonlinear modes, and consequently the reduction basis obtained is not suitable to the study of non-cyclic systems, such as the mistuned bladed-disk tackled in Section 4. By using a substructuring approach, the method proposed in the present paper does not require any symmetry assumption, and is thus more flexible and applicable to a broader range of nonlinear structures with no particular symmetry properties. The fixed-interface CMS method used here to perform the substructuring can however result in a large number of linear DOFs, which is why the nonlinear complex modes of the reduction basis – required to reduce the nonlinear DOFs – are used in combination with a set of linear vectors referred to as characteristic constraint modes [26] or interface modes [27,28] to reduce the remaining linear DOFs.

This paper is divided into three sections. Cornerstone of the method, the notion of nonlinear complex modes and their frequency-based computation are first reminded. The three steps of the reduced-order modelling technique are then derived, followed by the multiharmonic recast of the ROM and the aforementionned spectral substitution used to approximate the nonlinear forces. Finally, the method is applied on a three dimensional FE model of bladed disk subjected to dry friction nonlinearities and structural mistuning, which highlights the potential of the proposed method in dealing with real-life engineering structures.

2. Nonlinear complex modes

In this work, the definition of a nonlinear mode is that proposed by Laxalde and Thouverez in [15], and later used in several papers [24,25,29–31]. This definition comes with a very flexible computation procedure inspired by the HBM, allowing to tackle a broad range of systems without any restriction as to the intensity, smoothness, and dissipation of the nonlinear effects. This procedure is briefly reminded hereunder. Further information and examples can be found in the aforementioned papers.

Let a dynamical system be governed by the following set of ordinary differential equations (ODE),

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{g}(t) \quad (1)$$

with $\mathbf{M}, \mathbf{C}, \mathbf{K}$ the mass, viscous damping, and linear stiffness matrices, respectively, $\mathbf{g}(t)$ the external load vector, and $\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}})$ a nonlinear term from which may arise conservative as well as non-conservative forces. A nonlinear complex mode of this system refers to a pseudo-periodic solution of the underlying autonomous system. For practical reason, the autonomous system is here unaffected by viscous damping, but the procedure is perfectly applicable when retaining the damping matrix. The system to solve is

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{0} \quad (2)$$

Due to the non-conservative effects that may arise from the nonlinear term, the solution is sought has a decaying multi-harmonic oscillation

$$\mathbf{x}(t) = \mathrm{Re}\left\{ \sum_{k \in \mathbb{N}} \mathbf{x}_k e^{k\lambda t} \right\} \quad (3)$$

where λ is the complex eigenvalue of the mode. This expression is then substituted into the autonomous system and the orthogonality of the residual with respect to the basis functions $e^{k\lambda t}$ is enforced by means of the inner product

$$\langle f | g \rangle = \frac{2}{T} \int_0^T f(t)\overline{g(t)} dt \quad (4)$$

with T the period of oscillation. The orthogonality of the basis functions can be taken advantage of by neglecting the decay of the solution over the period used to enforce the orthogonality condition, as thoroughly explained in [15]. Practically speaking, the real part of λ is neglected in the exponential functions below the integral sign of the inner product. This assumption is not equivalent to neglecting the decay of the modes, and has been validated in numerous papers dealing with nonlinear complex modes [15,24,25]. The procedure yields a set of nonlinear and algebraic eigenproblems,

$$\left((k\lambda)^2 \mathbf{M} + \mathbf{K} \right) \mathbf{x}_k + \langle \mathbf{f}(\ldots, \mathbf{x}_k, \ldots, \lambda) | e_k \rangle = \mathbf{0} \quad \forall k \quad (5)$$

where e_k stands for the basis function of kth order. These equations are still coupled for all harmonic order through the nonlinear term, and must be solved simultaneously, which can be easily achieved by classic Newton-like solvers. It should be mentioned that the system is underdetermined, and must be supplemented by two equations prior to starting the solver. A basic *phase condition* together with a sequential continuation on a *control coordinate* q are used here as suggested in [15]. This control coordinate is used to normalise the components $\mathbf{x}_k$, so as to yield what will be referred to as nonlinear eigenvectors $\boldsymbol{\varphi}_k$,

$$\mathbf{x}_k = \boldsymbol{\varphi}_k q \quad (6)$$

As a consequence of the continuation scheme, the nonlinear mode, complex when non-conservative terms are effective, refers to the set of eigenvectors $\boldsymbol{\varphi}_k(q)$ and the corresponding eigenvalue $\lambda(q)$, known as functions of the control coordinate q.

3. Triple nonlinear modal synthesis with spectral substitution

This section introduces the theory of the reduced-order modelling technique presented in this paper. The method can be divided into three reduction steps, combined to a spectral substitution procedure allowing to fully take advantage of the nonlinear modes. The method relies on a substructuring approach [6] in order to provide both versatility and computational efficiency. It should be reminded that the second step of the procedure and the spectral substitution were already tested and validated in a previous work [25], which has led to the development of the method presented here so as to reveal its full potential on industrial structures.

3.1. First reduction – fixed-interface CMS

The first step of the procedure consists in building N linear super-elements by means of a standard Craig-Bampton fixed-interface CMS [32] retaining the boundary DOFs and the nonlinear DOFs as master coordinates. The objective of this first reduction step is twofold. First, it allows to reduce the computation time of the nonlinear complex modes in Section 3.2. Second, it makes