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A hybrid boundary element-statistical energy analysis for the mid-frequency vibration of vibro-acoustic systems

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ABSTRACT

Based on the concept of hybrid Finite Element (FE) analysis and Statistical Energy Analysis (SEA), a new hybrid method is developed for the mid-frequency vibration of vibro-acoustic systems. The Boundary Element (BE) method is used to describe the motion of a deterministic acoustic cavity. By enforcing the continuity conditions of displacement and velocity at the coupling interface, the dynamic coupling between the deterministic acoustic cavity and the statistical structure described by SEA is established. Then, a hybrid BE-SEA method for the mid-frequency vibration of vibro-acoustic systems is proposed. Post-processing provides formulations for calculating the sound pressure at points inside the acoustic cavity. Due to the nature of the BE method for acoustics, the proposed method not only has few degrees of freedom, but also automatically satisfies the Sommerfeld radiation condition at infinity for exterior acoustics problems. A numerical example compares results from the proposed hybrid BE-SEA method with those from the hybrid FE-SEA method and Monte Carlo simulation. The comparison illustrates that the proposed method gives good predictions for the mid-frequency behavior of vibro-acoustic systems and has the fewest degrees of freedom.

1. Introduction

Vehicles such as carrier rockets, aircraft and automobiles may be subjected to a wide range of excitation frequencies during their operation. The structure and the acoustic cavities around it form a typical complex vibro-acoustic system for which it is necessary to consider the coupling interaction between the structure and the acoustic cavities. In order to improve the safety, comfort and stability of the system it is essential to study the dynamic behavior of the complex vibro-acoustic system when optimizing the design.

A complex vibro-acoustic system will typically exhibit mixed mid-frequency behavior when it is excited in a mid-frequency environment. In general, it may be difficult to study the dynamic response of complex vibro-acoustic systems by using a deterministic method such as Finite Element (FE) analysis [1,2]. Since an appropriate element size (typically six to eight elements per wavelength [3,4]) is required to capture the detailed deformations of the system, the degrees of freedom may increase significantly with decrease of the wavelength as the frequency increases. To overcome this difficulty, many improved methods, such as higher-order techniques [5–7], reduction techniques [8–10] and the ultra-weak variational formulation [11], have been developed based on FE analysis. However, such element-based techniques are appropriate mainly for the dynamic analysis of systems at lower frequencies. In contrast, wave based approaches can provide complete deterministic analyses. Langley [12] and Bercin and Langley [13] obtained the forced vibration responses of complex systems consisting of rectangular plates by using the wave dynamic stiffness method. The energy flow was analyzed by Wester and Mace [14] using a wave method in which the flow of energy between components is described in terms of generalized 'wave components'. Ma et al. [15] converted the governing differential equations for transverse vibration of thin plates into Hamiltonian canonical equations, and then proposed a semi-analytical method for steady-state forced vibration response of rectangular thin plates. Barbarulo et al. [16] developed a technique, which combines the Variational Theory of Complex Rays proposed by Ladevèze and Arnaud [17] with proper generalized decomposition, for calculating the response of acoustic systems. Desmet [18] proposed the Wave Based Method (WBM) for the steady-state dynamic analysis of vibro-acoustic systems. Higher accuracy and efficiency can be provided by these wave-based methods for predicting the system response.

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However, uncertainties in the dimensions of the geometry and in the material properties inevitably arise during the manufacture and assembly of systems. As the frequency increases, the response of a system may be very sensitive to uncertainties, such as small imperfections in the system. Hence, it is necessary to consider the uncertainty of the system when it is excited in mid- and high-frequency ranges. However Monte Carlo simulation, in which both element-based and wave-based techniques can be employed to consider the uncertainty of the system, requires many reanalyses and is computationally expensive [19]. A popular statistical method is Statistical Energy Analysis (SEA) [20], in which the system is divided into a number of subsystems according to groups of similar modes. SEA employs the time, frequency and space average energy responses of each subsystem as the degrees of freedom, and establishes the power balance equation by considering the power exchange between the subsystems. A good prediction for the statistical behavior of the system at higher frequencies may be obtained by using SEA with little time cost. However, some assumptions in SEA may only be satisfied when the system has a high modal density at higher frequencies.

Unlike SEA, the statistical modal energy distribution analysis proposed by Maxit and Guyader [21,22] in the framework of SEA focuses on the power exchange between the modes, and not between whole subsystems. Statistical modal energy distribution analysis can be used even if the system has a low modal density, because it considers all resonant and non-resonant modes in the frequency range of interest, i.e. not just the resonant modes of the subsystem. MODal ENergy Analysis, developed by Totaro and Guyader [23] and based on the concept of the statistical modal energy distribution analysis, can provide energy analysis of subsystems at pure tone. The above two methods make a link between the energy methods and the FE method. The modal power balance equation can be established by modal analysis of the components in which the FE method models are of smaller simpler components, rather than the vibro-acoustic system as a whole. Langley [24] proposed the wave intensity method for analyzing the bending vibrations of a panel array at higher frequencies. This method provided a significant improvement over SEA by considering the directional filtering effects of the connections, which may lead to lower statistical wavefields. Langley and Bercin [25] then extended this method to the bending and in-plane vibrations of various plate structures. For a complex system, wave intensity techniques may face some difficulty when handling complex structural joints. The energy FE method [26,27] was developed to predict the average response of complex systems at high frequency ranges. The balance equation of the energy density is established by using net energy flow and energy superposition principles. Then, the equation can be solved using the FE method by considering the power flow between structural elements. The energy FE method can provide the local responses of the system. However the joint matrix, which was developed to deal with the discontinuity of the structures, may be difficult to obtain for complex junctions. Based on approaches describing essentially ray tracing type models [28], Le Bot [29] proposed a vibro-acoustic model for medium and high frequency analysis considering energy conservation of the systems. This model considers local variables, and can predict the repartition of energy density inside each subsystem. Tanner [30] proposed dynamical energy analysis for high frequency analysis of the vibro-acoustic systems, which interpolates between standard SEA and full ray tracing containing both these methods as limiting cases. According to this method, the typical SEA assumptions can be quantified in terms of the properties of the ray dynamics.

When a complex vibro-acoustic system is subjected to a mid-frequency excitation force, some components are subjected to short wavelength deformation, while other components are subjected to long wavelength deformation. Considering the different vibration behaviors of components, Zhao and Vlahopoulos [31] proposed a hybrid method, which combines the FE method and the energy FE method, for analyzing co-linear beam systems. The hybrid equations of the FE subsystems and energy FE subsystems, together with the interface equations, are solved simultaneously by an iterative process. Shorter and Langley [32,33] later presented a hybrid method combining FE and SEA for predicting the ensemble average response of complex vibro-acoustic systems. In this hybrid FE-SEA method, the FE method is employed to describe the motion of the so-called deterministic subsystems, which are subjected to long wavelength deformation, while the SEA is employed to model the so-called statistical subsystems, which are subjected to short wavelength deformation. The equations of the two types of subsystem are coupled by the diffuse-field reciprocity principle, which is a non-iterative relationship between the energy of the statistical subsystem and the cross-spectrum of the blocked reverberant force [33]. Based on the framework of hybrid FE-SEA method, Zhu et al. [34] developed the hybrid FE-energy FE method for analyzing beam-plate systems in the high frequency range. Ma et al. [35] developed a hybrid approach which employs the wave propagation method to describe the motion of rectangular plates. This method provides a good prediction for mid-frequency vibration analysis of built-up plate systems so long as the deterministic plates are rectangular. Langley and Cordioli [36] fully discussed the application of the hybrid FE-SEA method in the mid-frequency vibration analysis of vibro-acoustic systems with domain coupling of statistical subsystems, and proposed a reduced form of the deterministic subsystem equations. However, the FE method may face some difficulty when the acoustic cavity domain is unbounded. In the hybrid method proposed by Vergote et al. [37], the WBM was adopted to model the acoustic cavity instead of the FE method, so increasing computational efficiency.

The present paper develops a hybrid approach for the mid-frequency vibration of vibro-acoustic systems within the framework of the hybrid FE-SEA method. Based on the concept of hybrid FE-SEA, the proposed method adopts the Boundary Element (BE) method [38,39] to describe the motion of the acoustic cavity instead of the FE method or WBM. As is well known, the BE method is a powerful method for acoustics. Due to the nature of the BE method, the proposed method only discretizes the boundary of the acoustic cavity into elements, which is easier than discretizing the acoustic cavity domain and leads to a smaller number of degrees of freedom. It is also better able to handle exterior acoustics problems. Section 2 outlines the basic principles of the hybrid method. Section 3 derives the governing equation and power balance equation, followed by formulations for calculating the sound pressure at points inside the acoustic cavity. The numerical example in Section 4 validates the proposed hybrid BE-SEA method against the hybrid FE-SEA and Monte Carlo simulation.

2. Basic principles of the hybrid method

In the hybrid FE-SEA method proposed by Shorter and Langley [32], a complex system is modeled as an assembly of deterministic and statistical subsystems according to deformation wavelengths. The deterministic components are modeled using the FE method, while the statistical components are modeled as SEA subsystems. The response of statistical components is viewed as the superposition of two wave fields, one of which is the direct field formed by the initial generated waves, prior to any boundary reflections, and the other is the reverberant field formed by the waves produced on the first and all subsequent reflections [33].

The presence of uncertainties in the statistical system is accounted for in the reverberant field, with the result that the direct field is deterministic across the ensemble. The direct field