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Application of the edge-based gradient smoothing technique to acoustic radiation and acoustic scattering from rigid and elastic structures in two dimensions

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A B S T R A C T

As is well-known to all, the conventional finite element method (FEM) is constrained by the "numerical dispersion error" issue for solving acoustic problems at high frequencies. In this paper, the gradient smoothing technique (GST) which is based on the edges of the elements is combined with the conventional FEM to construct a novel edge-based smoothed FEM (ES-FEM) for two dimensional exterior structural–acoustic problems. The smoothed gradient field is obtained by performing the GST over the obtained smoothing domains (SDs). The present ES-FEM is able to provide a relatively appropriate stiffness of the real system owing to the "softening effects" from the GST. Therefore, the accuracy of the numerical solutions can be significantly improved. To effectively handle the exterior Helmholtz equation in unbounded domains, a predefined artificial boundary B is employed to obtain a bounded computational domain and the well-known Dirichlet-to-Neumann (DtN) map is used to prevent the possible reflections from the far-field. Several supporting numerical examples indicated that the ES-FEM with DtN map was very effective for exterior structural–acoustic problems and could produce more accurate numerical results than the conventional FEM.

1. Introduction

The acoustic radiation and acoustic scattering from rigid and elastic structures are often encountered in practical engineering applications. Therefore, how to effectively compute and predict the acoustic radiated and the acoustic scattered field is very important to improve the acoustical properties of the engineering structures. In the coupled exterior structural-acoustic system (acoustic radiation and acoustic scattering) in the unbounded domains, if the acoustic medium around the submerged structure is light fluid (such as air), the structure-acoustic coupling effects can be neglected and the submerged structure can be considered as an ideal rigid object. However, if the acoustic medium around the submerged object is heavy fluid (such as water), the assumption which is frequently made that the surface of submerged objects is perfectly rigid is often an oversimplification of the true situation and the interaction between the structural field and the acoustic field should be considered. In general, the analytical or semi-analytical approaches [1–5] are only effective for the problems with simple geometries. When it comes to more practical problems with very complicated geometries, the analytical solutions are always impossible to obtain. In these cases, we have to resort to numerical methods.

In the past over 50 years, many numerical techniques have been proposed for solving the coupled structural-acoustic problems. In these numerical approaches, the finite element method (FEM) and the boundary element method (BEM) are two most versatile and powerful numerical techniques. The conventional BEM is essentially a boundary-based approach and only the boundary discretization is needed. Additionally, the Sommerfeld radiation condition is inherently satisfied in the BEM formulation. However, due to the non-local property of the BEM, the obtained BEM system matrices are always dense and non-banded. Recently, Nguyen et al. [6,7] incorporated the isogeometric analysis into the symmetric Galerkin BEM to give a novel IGA-SGBEM for two-dimensional crack and three-dimensional elasticity problems and it is found

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that the highly accurate numerical solutions could be obtained using this advanced BEM. In contrast to the classical BEM, the FEM is a typical domain-based method and it usually leads to sparse and banded matrices, which is helpful to reduce the computational cost and memory requirements. However, the FEM for acoustic problems also suffers from a few innate drawbacks.

The first is how to effectively handle the problems in infinite space. Since the standard FEM is based on finite domain rather than infinite domain, hence additional numerical techniques, such as the Dirichlet-to-Neumann (DtN) map [8] or the perfectly matched layer (PML) [9], are always required for FEM to solve the exterior acoustic problems. Although these methodologies can enable the classical FEM to solve the acoustic problems in infinite domains, they also more or less increase the difficulty and complexity of the FEM for the exterior acoustics.

Another important issue of the classical finite element model for acoustic computation is the dispersion error [10,11]. Owing to the well-known "overly-stiff" property of the classical FEM, the obtained numerical solutions for acoustic problems always suffer from the numerical dispersion error issue. The obtained wave number values from FEM are often smaller than the exact ones. Generally, in small wavenumber range the standard FEM is indeed effective in acoustic computation. However, in large wavenumber range it cannot provide sufficiently reliable solutions unless very refined mesh is used. Unfortunately, the use of refined mesh may also dramatically increase the computational cost. In practice, FEM researchers always follow the "rule of thumb" (at least six elements to divide a wavelength) [12] to retain the quality of the FEM solutions in relatively large wavenumber range. However, it is found that the numerical error cannot be removed completely and it will exist all the same whether this criterion is satisfied or not. For the purpose of enhancing the ability of the FEM scheme in handling the acoustic problems and controlling the numerical error more effectively, a profusion of modified FEMs, extended FEMs and meshfree methods have been proposed to handle this issue, such as the Galerkin/least-squares FEM (GLS-FEM) [12,13], the generalized FEM (GFEM) [14,15], the element-free Galerkin (EFG) method [16–18] and so on. Although these relatively novel numerical methods could reduce the numerical error to a certain extent, the numerical error in Helmholtz problems still cannot be suppressed entirely. So far there are still no ideal numerical techniques for acoustic computation and the quest for this will continue.

In recent years, Liu et al. developed a series of smoothed finite element methods (S-FEMs) [19–23], which is formulated from a novel Galerkin weakened weakform (or W^2) [24,25] rather than the standard Galerkin weakform in the standard FEM formulation, to deal with the linear elastic solid mechanics problems. In contrast to the point interpolation method (PIM) [25–27], which is a typical meshfree method, the S-FEM is developed by combining the gradient smoothing technique (GST) from the meshless techniques [28,29] with the classical FEM. Therefore, the S-FEM not only has good features from the FEM but also possesses some particular properties from the meshfree methods. In S-FEM, the smoothed gradient fields are obtained by performing the gradient smoothing operations over the predefined smoothing domains (SDs). From a large quantity of numerical examples, it is found that better numerical solutions could be usually obtained by the S-FEM compared with the original FEM. The possible reason for this might be that the "softening effect" provided by the GST can properly soften the "over-stiff" FEM model and the obtained S-FEM system stiffness is closer to the exact ones than the conventional FEM. Initially, the S-FEMs were employed to solve solid mechanics problems [30–36]. Due to their outstanding performance, then they had been applied to handle many other engineering problems, such as heat transfer problems [37,38], fracture mechanics problems [39–42], piezoelectric structures [43], plates and shells [44–54], fluid-structure interaction problems [55,56] and so on.

However, to the authors' best knowledge, the S-FEM has seldom been used to handle the acoustic radiation and acoustic scattering problems. As an extension of the previous work about S-FEM, the present paper presents the utilization of the S-FEM for modelling the exterior acoustic problems. From the numerical results in this paper, it is found that the ES-FEM could produce more accurate results than the standard FEM for the exterior structural-acoustic problems yet it is simple to implement. Therefore, compared with the standard FEM, the S-FEM might be a good alternative for exterior acoustics and has great potential application in engineering problems. Although both the basic idea of the ES-FEM and the used error estimate for acoustic problems are not originally proposed in this work, the present paper represents an attempt to quest the new numerical techniques to suppress the troublesome dispersion error in solving Helmholtz equation, which is still an unsolved issue in modern computational acoustics [57]. The present paper is structured as follows: Section 2 presents the fundamental theory formulation for the exterior acoustic problems. The detailed formulation of E-GST is given in Section 3. Section 4 includes the error estimates of the numerical solutions for Helmholtz problem. The coupled formulations for the general exterior structural-acoustic problems are presented in Section 5. A number of supporting numerical experiments and the related conclusions are then summarized in the remaining sections.

2. The standard Galerkin weakform for the exterior acoustics

A typical exterior acoustic problem is shown in Fig. 1, let R be an infinite domain with homogeneous isotropic medium and the obstacle with piecewise smooth boundary is embedded in this unbounded domain. The boundary of the obstacle is denoted by Γ and it admits the following partition: $\Gamma = \Gamma_v \cup \Gamma_p$ and $\Gamma_v \cap \Gamma_p = \emptyset$, where Γ_v and Γ_p are the Neumann boundary condition and Dirichlet boundary condition, respectively.

The acoustic pressure p in the above-mentioned exterior boundary-value problem should satisfy the following equations:

$$\Delta p + k^2 p + f = 0 \quad \text{in } R \tag{1}$$

$$p = g \quad \text{on} \quad \Gamma_p \tag{2}$$

$$\nabla p \cdot n = l \quad \text{on} \quad \Gamma_v \tag{3}$$

where Eq. (1) is the well-known reduced wave equation (or Helmholtz equation); Δ is the Laplace operator; k denotes the wavenum-

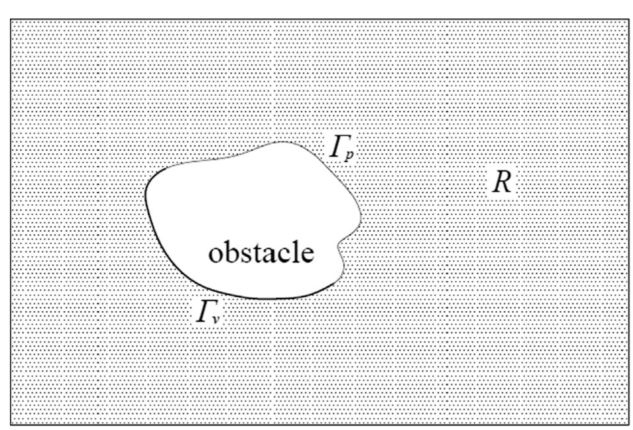

Fig. 1. An obstacle with smooth boundary immersed in the infinite problem domain.