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A weak form quadrature element formulation for geometrically exact thin shell analysis

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ABSTRACT

The present paper addresses a weak form quadrature element formulation for the geometrically exact thin shell model in which the Kirchhoff-Love hypothesis is adopted. The displacement derivative continuity conditions are enforced by the reconstruction of rotation variables at the edges of elements. By the utilization of rotation quaternions, a total Lagrange updating scheme is implemented for edge constraint director rotations. Several numerical examples are presented to illustrate the effectiveness of the proposed formulation and the significant reduction in the number of degrees of freedom in geometrically nonlinear thin shell analysis with large displacements and rotations.

1. Introduction

The Mindlin-Reissner and the Kirchhoff-Love hypotheses are well-known for the classical theory of shells. The former incorporates first-order transverse shear deformation, making it feasible to describe moderately thick shells. By contrast, the latter only takes membrane and bending deformation into account, assuming that the transverse shear deformation is negligible, which is fit for simulating thin shells [1]. The absence of shear strains implies that the cross-section is perpendicular to the tangent plane of the neutral surface according to the Kirchhoff-Love shell hypothesis, enabling the rotation parameters to be related to the first-order spatial derivatives of displacements. It can be seen that such a relation implies the implementation of the C^1 continuity requirement. In many cases these requirements are not so easy to be satisfied, especially in geometrically nonlinear shell analysis, where the relationships between rotations and displacement derivatives are complex. This may be one reason for the higher popularity of Mindlin-Reissner shell hypothesis in the finite element schemes, although thin shell structures are common in most problems.

Among the majority of the research work on numerical implementations for geometrically nonlinear shell under Mindlin-Reissner hypothesis, the geometrically exact shell model proposed by Simo [2,3], as an analogue of the geometrically exact beam model [4,5], has a profound influence in the past decades. By utilizing the Cosserat theory [6], the geometrically exact model can easily handle large displacements and rotations in structure deformation. Further studies by Simo and other researchers have tremendously broadened the feasibility of this model in a variety of analyses [7–10]. The mathematical representation and updating schemes of spatial rotation are crucial issues in implementations of geometrically exact model. The quaternion representation avoids the singularity problems in describing large total rotation, thus providing a convenient tool for rotation formulation of geometrically exact model. The adoption of quaternion in geometrically exact beam model was firstly introduced by Vu-Quoc and Simo [5,11]. A detailed discussion of this issue and a reformation of rotation terms in the realm of Clifford algebra were given by Mcrobie and Lasenby [12]. For classical shell models undergoing finite rotation, constraint rotation vectors with different updating schemes [3,13] are commonly utilized for the exclusion of drilling rotation. A comprehensive introduction of the interrelations between different parametrizations for rotation updating in geometrically exact shell analysis was presented by Brank and Ibrahimbegović [14].

In low-order finite element implementations of Mindlin-Reissner shells, a major drawback is the shear locking phenomena caused by independent interpolation of displacements and rotations, which may incur poor results, especially for thin shell structures. By contrast, the Kirchhoff-Love shell model using displacements as the only unknowns can bypass the shear locking problem without extra remedy and avoid dealing with complex spatial rotations, which is appealing to researchers. In addition, the challenge of implementing displacement derivative continuity condition also makes this model a benchmark problem for new numerical developments. Krysl and Belytschko proposed a

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meshless approach to analyzing thin shells [15], which was further improved later to model cracks and incorporate finite strains [16]. Cirak et al. introduced the subdivision surface method on the basis of the spline approximation of configuration to develop a practical finite element formulation for thin shells [17] and extended it to non-manifold structures [18]. Kiendl et al. derived a Kirchhoff-Love shell element on the basis of the isogeometric concept and showed its advantage on geometric representation [19]. Noels presented a discontinuous Galerkin formulation to weakly enforce the continuity of displacement derivatives between thin shell elements [20]. Millán et al. adopted maximum entropy meshfree approximation in manifold description of thin shells [21]. Ivannikov et al. discussed the question on the imposition of kinematic boundary conditions for the nonlinear Kirchhoff-Love shells and proposed a set of parameters for boundary rotations under different conditions [22].

The weak form quadrature element method (abbreviated as QEM) is a numerical scheme that combines the differential quadrature analogue and numerical integration schemes to approximate and discretize the variational description of a problem [23,24]. There are a variety of weak form quadrature elements, depending on the choice of the numerical integration scheme. When efficient numerical schemes such as Lobatto quadrature are adopted, the differential quadrature analogue [25,26] embodies the essential ideas of the pseudo-spectral method [27] and the QEM overlaps with the spectral element methods [28–30] accordingly. Due to the coincidence of integration points and interpolation nodes within elements, the QEM exhibits advantages in representing configurations with high-order approximants and circumventing some problems (e.g. the loss of objectivity in large deformation analysis) brought by improper interpolation between nodes and integration points. Besides its high efficiency, it is feasible to overcome shear and membrane locking problems because of its high-order characteristics that effectively alleviate the influences of constraints incurred by couplings of different order approximations to bring about reconciliation of deformation [23,31]. Weak form quadrature element formulations have been successfully implemented for the analysis of nonlinear beams and shells on the basis of relevant geometrically exact models in previous investigations [31–33].

Analogous to the previous study on Simo's geometrically exact Mindlin-Reissner shell model [31], a weak form quadrature element formulation of the geometrically exact Kirchhoff-Love shell is presented in this paper. Compared with the previous formulation [31], the number of degrees of freedom (DOF) is significantly reduced in the present formulation by the elimination of inner rotations. The shear-rigid hypothesis of thin shell is inherently incorporated here. Differing from the previous C^0 continuity quadrature element, a new scheme for weak form quadrature shell element to meet the C^1 continuity requirement is proposed. The element edge tangent rotation variables are reconstructed by introducing edge constraint directors for the convenience of enforcing rotational continuity conditions, circumventing the cumbersome use of displacement derivative parameters. The rotation quaternion is utilized for the expression of rotations of edge constraint directors, thus making it feasible to implement a total Lagrange updating scheme for the variables in the shell model, which avoids the accumulation of computational errors.

The remaining part of this paper is organized as follows. In Section 2, the basic ideas and relevant equations of the geometrically exact Kirchhoff-Love shell are given. The weak form quadrature element formulation is established in Section 3. Six numerical examples are presented in Section 4 to evaluate the performance of the formulation. Conclusions are drawn in Section 5.

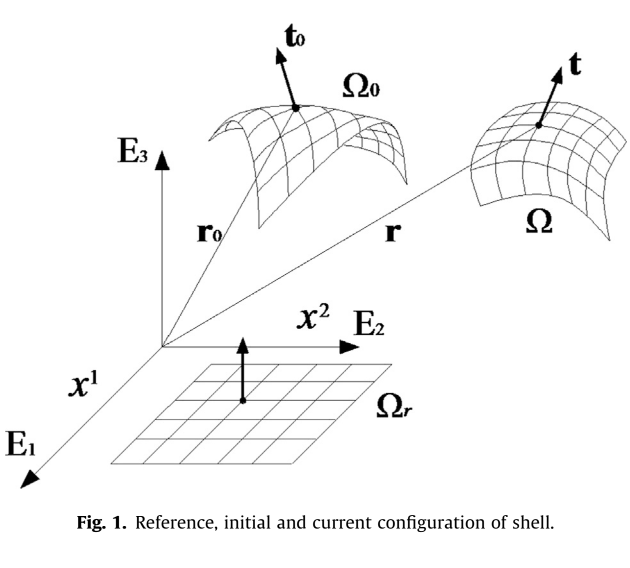

Fig. 1. Reference, initial and current configuration of shell.

2. Geometrically exact Kirchhoff-Love shell

2.1. Configuration description and kinematic assumption

As shown in Fig. 1, the reference configuration, the initial configuration and the current configuration of a shell described by Simo's geometrically exact model are denoted by Ω_r, Ω_0 and Ω, respectively. The Cartesian frame $\{\mathrm{E}_i\}(i = 1, 2, 3)$ serves as the reference coordinate system. It can be seen that the configuration is characterized by the position vector $\mathbf{r}$ that determines the mid-surface and a corresponding unit normal vector $\mathbf{t}$. The position of an arbitrary material point in the current configuration Ω_r can be expressed as

$$\boldsymbol{\phi} = \mathbf{r}(x^\alpha) + x^3\mathbf{t}(x^\alpha), \quad x^3 \in [h^-,\ h^+]. \tag{1}$$

The convected coordinates x^α describe the configuration that is uniquely determined by $\mathbf{r}$ and $\mathbf{t}$, while coordinate x^3 locates the position along the thickness direction of the shell. The Greek indices range from 1 to 2 in the present paper. The thickness of the shell $h = h^+ - h^-$ is assumed to be invariant during deformation. For finite deformation analysis, additional parameters are needed to incorporate thickness stretch [34], the corresponding derivation is straightforward and is not discussed in this paper accordingly. Similarly, the material point in the initial configuration Ω_0 is given by

$$\boldsymbol{\phi}_0 = \mathbf{r}_0(x^\alpha) + x^3\mathbf{t}_0(x^\alpha), \quad x^3 \in [h^-,\ h^+]. \tag{2}$$

From Eqs. (1) and (2), the initial and current local covariant frames attached to the mid-surface of the shell are defined as

$$\{\, \mathbf{g}_1 \quad \mathbf{g}_2 \quad \mathbf{t}_0 \,\} = \{\, \mathbf{r}_{0,1} \quad \mathbf{r}_{0,2} \quad \mathbf{t}_0 \,\} \tag{3}$$

and

$$\{\, \mathbf{t}_1 \quad \mathbf{t}_2 \quad \mathbf{t} \,\} = \{\, \mathbf{r}_{,1} \quad \mathbf{r}_{,2} \quad \mathbf{t} \,\}, \tag{4}$$

respectively; the comma denotes partial differentiation, i.e. $(\bullet)_{,\alpha} = \partial(\bullet)/\partial x^\alpha$. The covariant metric tensor $g_{\alpha\beta}$ is defined by the initial covariant frame as

$$g_{\alpha\beta} = \mathbf{g}_\alpha \bullet \mathbf{g}_\beta, \tag{5}$$

and the contravariant metric tensor $g^{\alpha\beta}$ is obtained by

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma, \tag{6}$$

where δ^α_γ is the Kronecker delta function.

For Kirchhoff-Love shells, the unit vector $\mathbf{t}$ coincides with the normal of the mid-surface of the shell, namely