Computers and Structures 202 (2018) 60–73

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Optimization of a class of composite method for structural dynamics

H.M. Zhang, Y.F. Xing *

Institute of Solid Mechanics, Beihang University, Beijing 100083, China

ARTICLE INFO

Article history:
Received 28 November 2017
Accepted 15 March 2018

Keywords:
Composite
Optimal
Strong numerical damping
Higher accuracy

ABSTRACT

A class of composite method that implements the trapezoidal rule in the first few sub-steps and the backward difference formula in the last sub-step is studied in this paper. The optimal schemes of two sub-step, three sub-step and four sub-step methods, where the four sub-step composite scheme is developed for the first time, are proposed by optimizing their accuracy. Compared with several existing composite methods, the optimal schemes are also endowed with second-order accuracy, unconditional stability and strong numerical damping, and they can achieve higher amplitude and period accuracy under the same amount of calculation. Moreover, it follows that in the optimal schemes the more sub-steps the higher accuracy, so the optimal four sub-step method is highly recommended. Several test problems are used to validate the performance.

1. Introduction

Time integration methods include explicit and implicit methods. Explicit methods cost less per step, while implicit methods are more competitive due to the permission of larger time step. In implicit methods, numerical dissipation is important to remove the spurious high-frequency modes introduced by spatial discretization, and benefits numerical stability of the method when applied to nonlinear systems. In this study we focus on the composite implicit methods, so several kinds of representative implicit methods are first reviewed below.

Following the Newmark method [1], a series of single-step collocation methods [2–6] show second-order accuracy, unconditional stability and controllable numerical damping. And the energy conserving and decaying methods [7–11] can meet the energy criterion for nonlinear systems. These two kinds of methods are good candidates for structural dynamics, but improving their dissipation capability is accompanied by lowering their accuracy.

The Bathe method [12–14] is a two sub-step composite method using the trapezoidal rule in the first sub-step and the three-point Euler backward formula in the second sub-step, and was demonstrated to be reliable and effective for linear and nonlinear dynamics. The composite method combines the advantages of the employed trapezoidal rule and three-point Euler backward formula, where the former improves low-frequency accuracy, and the latter improves high-frequency dissipation. Further investigations [15–18] have checked the performance of the Bathe method on structural dynamics and wave propagation problem, and optimized the sub-step division.

Afterwards, several implicit composite methods [19–22] involving three or four sub-steps were developed. A combination of the trapezoidal rule and the higher-order Newton backward extrapolation functions yields the multi-sub-step higher-order implicit time integration family [19]. However, this kind of methods of more than four sub-steps are unstable and more than two sub-steps are conditionally stable. Chandra et al. [21] proposed a three-sub-step scheme using the trapezoidal rule in the first two sub-steps and the backward difference formula in the last sub-step, named the TTBDF method (the Trapezoidal rule–the Trapezoidal rule–the Backward Difference Formula). Besides, another three-sub-step method [22], the Wen method, employed the trapezoidal rule, the backward difference formula and the Houbolt method in the first, second and third sub-step, respectively. These two three-sub-step methods show similar attributes to the Bathe method, but the comparative study [23] indicated that the TTBDF method possesses higher accuracy, while the Wen method [22] is more suitable for solving wave propagation problem.

In order to achieve better performance, the present authors propose a four sub-step scheme in this work, referred to as the TTTBDF method (the Trapezoidal rule–the Trapezoidal rule–the Trapezoidal rule–the Backward Difference Formula), and the class of composite methods that adopt the trapezoidal rule in the first few sub-steps and the backward difference formula in the last sub-step are optimized. Compared with the existing composite methods, the optimal schemes of the TTBDF and TTTBDF methods, referred to as the OTTBDF and OTTTBDF methods respectively, are more accurate and provide sufficient high-frequency dissipation as

* Corresponding author.
E-mail address: xingyf@buaa.edu.cn (Y.F. Xing).

https://doi.org/10.1016/j.compstruc.2018.03.006
0045-7949/

well under the same amount of calculation. Since theoretical analysis is limited to the linear range, the pendulum problem is employed to check the performance on nonlinear dynamics and the results illustrate that the proposed optimal schemes possess desirable stability and accuracy.

This paper is organized as follows. In Section 2, the basic formulations for the two, three and four sub-step schemes are presented and the optimal parameters are generated by linear analysis. Then the comparative study on properties of the optimal schemes and several existing composite methods is conducted in Section 3. The numerical simulations including linear and nonlinear problems are implemented in Section 4. Finally, the conclusions are drawn in Section 5.

2. Formulations

The generalization of the composite methods by applying the trapezoidal rule in the first few sub-steps and the backward difference formula in the last sub-step was firstly proposed in Ref. [12]; however, in this class, the current multi-sub-step methods mostly dividing the step into equal parts are not the optimal schemes.

In this section, the Bathe method and the TTBDF method are reviewed, and the TTTBDF method is formulated. On the basis of second-order accuracy and unconditional stability, the optimal schemes are generated by the measures of percentage amplitude decay and period elongation. In the TTBDF and TTTBDF methods, several factors such as computation cost and matrix storage are taken into account to provide additional parameter relations.

2.1. The Bathe method

In the Bathe method, the time interval $[t, t+h]$ is divided into two sub-steps $[t,\ t+\gamma h]$ and $[t+\gamma h,\ t+h]$, where $0<\gamma<1$ is an adjustable parameter. In the first sub-step, the trapezoidal rule is used as

$$\begin{aligned} \boldsymbol{x}_{t+\gamma h} &= \boldsymbol{x}_t + \frac{\gamma h}{2}(\dot{\boldsymbol{x}}_t + \dot{\boldsymbol{x}}_{t+\gamma h}) \\ \dot{\boldsymbol{x}}_{t+\gamma h} &= \dot{\boldsymbol{x}}_t + \frac{\gamma h}{2}(\ddot{\boldsymbol{x}}_t + \ddot{\boldsymbol{x}}_{t+\gamma h}) \end{aligned} \tag{1}$$

In the second sub-step, the three-point Euler backward method is employed as

$$\begin{aligned} h\dot{\boldsymbol{x}}_{t+h} &= \theta_2 \boldsymbol{x}_{t+h} + \theta_1 \boldsymbol{x}_{t+\gamma h} + \theta_0 \boldsymbol{x}_t \\ h\ddot{\boldsymbol{x}}_{t+h} &= \theta_2 \dot{\boldsymbol{x}}_{t+h} + \theta_1 \dot{\boldsymbol{x}}_{t+\gamma h} + \theta_0 \dot{\boldsymbol{x}}_t \end{aligned} \tag{2}$$

where the parameters satisfy

$$\theta_2 = \frac{\gamma-2}{\gamma-1}, \quad \theta_1 = \frac{1}{\gamma(\gamma-1)}, \quad \theta_0 = -\frac{\gamma-1}{\gamma} \tag{3}$$

For linear structural system, the equilibrium equations used in two sub-steps are respectively as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}_{t+\gamma h} + \boldsymbol{C}\dot{\boldsymbol{x}}_{t+\gamma h} + \boldsymbol{K}\boldsymbol{x}_{t+\gamma h} = \boldsymbol{R}_{t+\gamma h} \tag{4}$$

$$\boldsymbol{M}\ddot{\boldsymbol{x}}_{t+h} + \boldsymbol{C}\dot{\boldsymbol{x}}_{t+h} + \boldsymbol{K}\boldsymbol{x}_{t+h} = \boldsymbol{R}_{t+h} \tag{5}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ and $\boldsymbol{R}$ are the mass matrix, damping matrix, stiffness matrix and external load vector, respectively; $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$ and $\ddot{\boldsymbol{x}}$ are the displacement, velocity and acceleration, respectively.

In terms of Eqs. (1)–(5), the time-stepping equations can be written as

$$\widehat{\boldsymbol{K}}_1 \boldsymbol{x}_{t+\gamma h} = \widehat{\boldsymbol{R}}_1 \tag{6}$$

$$\widehat{\boldsymbol{K}}_2 \boldsymbol{x}_{t+h} = \widehat{\boldsymbol{R}}_2 \tag{7}$$

where the effective stiffness matrices and load vectors are

$$\widehat{\boldsymbol{K}}_1 = \frac{4}{\gamma^2 h^2}\boldsymbol{M} + \frac{2}{\gamma h}\boldsymbol{C} + \boldsymbol{K} \tag{8}$$

$$\widehat{\boldsymbol{R}}_1 = \boldsymbol{R}_{t+\gamma h} + \boldsymbol{M}\left(\frac{4}{\gamma^2 h^2}\boldsymbol{x}_t + \frac{4}{\gamma h}\dot{\boldsymbol{x}}_t + \ddot{\boldsymbol{x}}_t\right) + \boldsymbol{C}\left(\frac{2}{\gamma h}\boldsymbol{x}_t + \dot{\boldsymbol{x}}_t\right) \tag{9}$$

$$\widehat{\boldsymbol{K}}_2 = \frac{\theta_2^2}{h^2}\boldsymbol{M} + \frac{\theta_2}{h}\boldsymbol{C} + \boldsymbol{K} \tag{10}$$

$$\begin{aligned} \widehat{\boldsymbol{R}}_2 = \boldsymbol{R}_{t+h} &- \boldsymbol{M}\left(\frac{\theta_2}{h^2}(\theta_1 \boldsymbol{x}_{t+\gamma h} + \theta_0 \boldsymbol{x}_t) + \frac{1}{h}(\theta_1 \dot{\boldsymbol{x}}_{t+\gamma h} + \theta_0 \dot{\boldsymbol{x}}_t)\right) \\ &- \boldsymbol{C}\left(\frac{1}{h}(\theta_1 \boldsymbol{x}_{t+\gamma h} + \theta_0 \boldsymbol{x}_t)\right) \end{aligned} \tag{11}$$

Then the velocity and acceleration are updated according to Eqs. (1) and (2). For linear systems, the effective stiffness matrices in Eqs. (8) and (10) are factorized prior to the recursion. However, in nonlinear analysis, the tangent stiffness matrix changes in every iteration.

The Bathe method is preferred since it can effectively preserve the low-frequency modes ($\rho \to 1$ when $\omega h \le 0.6$) and filter out the high-frequency modes ($\rho \to 0$ when $\omega h \ge 2$). Figs. 1 and 2 show the percentage amplitude decay and period elongation versus γ for several different τ ($\tau = \omega h$). It is observed that regardless of the value of τ, the extreme values of these curves occur at the same γ. The theoretical analysis yields the extreme point as

$$\gamma = 2 - \sqrt{2} \tag{12}$$

which features the minimum period elongation but the maximum amplitude decay. In addition, it also reduce computation cost for linear system since two sub-steps share the same effective stiffness matrix $4\boldsymbol{M}/\gamma^2 h^2 + 2\boldsymbol{C}/\gamma h + \boldsymbol{K}$ as $\theta_2 = 2/\gamma$. As a result, $\gamma = 2 - \sqrt{2}$ is the optimal parameter for the Bathe method. The same conclusion was also reached in Refs. [17,18] by a similar way.

2.2. The TTBDF method

The TTBDF method [21] divides the time step $[t, t+h]$ into three sub-steps $[t,\ t+\gamma_1 h]$, $[t+\gamma_1 h,\ t+\gamma_2 h]$ and $[t+\gamma_2 h,\ t+h]$ where $0<\gamma_1<\gamma_2<1$. The trapezoidal rule is employed in the first two sub-steps as

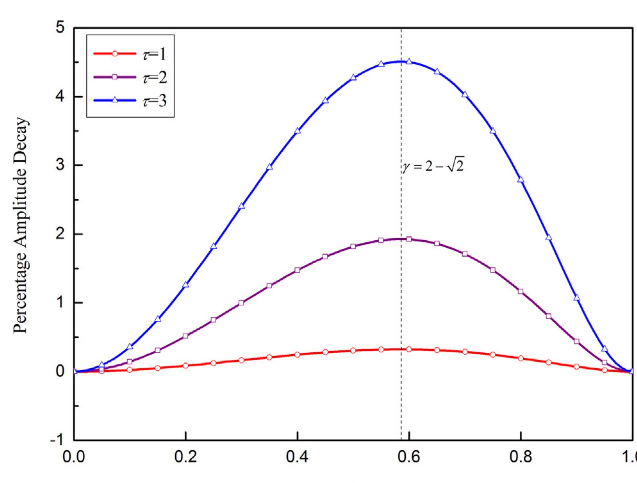

Fig. 1. Percentage amplitude decay versus γ for the Bathe method.