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A new 8-node element for analysis of three-dimensional solids

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ABSTRACT

We propose a new 8-node hexahedral element, the 3D-MITC8 element, for the analysis of three-dimensional solids. We use the MITC method and find the assumed strain field from a thought experiment using a truss idealization. For geometric nonlinear analysis, when needed to suppress hour-glass deformations, the formulation also uses automatically displacement-based contributions to the shear strains. The element shows a much better predictive capability than the displacement-based element. It is computationally more effective than the 8-node element with incompatible modes, and considering accuracy, in linear analysis performs almost as well, and in nonlinear analyses we do not observe spurious instabilities. We show that the new 3D solid element passes all basic tests (the isotropy, zero energy mode and patch tests) and present the finite element solutions of various benchmark problems to illustrate the solution accuracy reached with the new element.

1. Introduction

A three-dimensional 8-node hexahedral solid finite element is frequently employed for the finite element analysis of solids in engineering practice. The element can be used to model many three-dimensional (3D) solids and performs considerably better than the 4-node tetrahedral element. However, the standard 8-node 3D solid element does not satisfy the inf-sup conditions, hence the solution accuracy can severely deteriorate due to shear and volumetric locking [1,2].

To improve the behavior of the standard displacement-based element, additional "incompatible modes" are frequently used [3]. The incompatible modes technique is a special case of the enhanced assumed strain (EAS) method, and the resulting 8-node 3D solid element requires, compared to the standard pure displacement-based element, an additional 9 internal degrees of freedom to represent the conditions of pure bending [3–6]. The element is quite powerful since it alleviates both shear and volumetric locking, but it uses the additional degrees of freedom and can show a non-physical instability in the analyses of nonlinear problems [5–8].

The instability of the EAS elements has been observed to occur in both small and large strain nonlinear analyses [7–14]. If an element mesh is subjected to compression, a spurious hour-glass bending mode may occur in elements eventually resulting into an indefinite stiffness matrix at a certain critical compressive strain [8]. Initially, the hour-glass deformations are small but as they grow, the incremental analysis leads to a spurious collapse of the model.

To treat the spurious instability special solution methods and various element formulations have been developed. The variational principle for nonlinear analysis has been modified, stabilization parameters have been proposed, and mixed-enhanced elements have been developed [9–14], see these references and the references therein. However, further developments are of much interest and, based on our success of developing reliable and efficient shell elements based on the MITC technique [1], we believe that we can also obtain an effective 3D eight-node MITC element.

In this paper we propose a new 8-node hexahedral element based on the standard displacement interpolations and the MITC (Mixed Interpolation of Tensorial Components) approach [1,2,10,15–20]. To obtain a stable element, we choose the tying positions and strain interpolations based on the physical behavior of a simple truss structure that idealizes the 8-node solid element. For geometric nonlinear analysis to suppress hour-glass deformations, the formulation also uses automatically when needed a stabilization scheme based on displacement-based contributions in shear strains. Using these key ideas for the assumed strain field, we find the 3D-MITC8 element to give solution stability, good solution accuracy, and to be computationally efficient. We also extend this element to obtain the 3D-MITC8/1 element based on a mixed displacement-pressure formulation.

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In the next section we present the concepts we use for the tying and interpolation of the strains in the MITC procedure. Then, in Section 3, we propose the new 3D-MITC8 and 3D-MITC8/1 elements using the total Lagrangian (T.L.) formulation. Of course, the linear behavior corresponds to the first step in the T.L. formulation. Further, in Section 4, the stability and accuracy of the element are assessed using basic tests (the isotropy, zero energy mode and patch tests) and the solutions of various benchmark problems. Finally, in Section 5, we present our conclusions.

2. Tying and interpolation of strains in the MITC procedure

In this section, we present the concepts we employ in the MITC procedure for the new element. The tying positions and interpolations of the assumed strain components are developed considering stability in linear and nonlinear analyses.

The geometry of an 8-node hexahedral solid element is shown in Fig. 1. The element domain is given and the strain components are defined corresponding to the three natural coordinates, r, s and t. For the 3D solid element, there are three normal (in the directions of r, s and t) and three shear (on the planes of rs, st and tr) strain components. The six assumed strain components are denoted by ${}_0\tilde{e}_{rr}$, ${}_0\tilde{e}_{ss}$, ${}_0\tilde{e}_{tt}$, ${}_0\tilde{e}_{rs}$, ${}_0\tilde{e}_{st}$ and ${}_0\tilde{e}_{tr}$.

The choice of assumed strain interpolation must be such that the solid element is stable corresponding to each strain component. To obtain insight, we idealize the hexahedral domain as a truss structure with 8 joints that correspond to the nodes, see Fig. 2(a), of the 8-node 3D element. The selected truss structure is shown in Fig. 2(b). This structure is stable and consists of the minimum number of 2-node truss elements. We next consider the location and direction of each truss element to correspond to an assumed strain component. The location of tying is given by the truss element but to obtain better accuracy using the 3D element in analyses we can move these locations to corresponding Gauss integration points.

If the truss structure we use is stable with the minimum number of truss elements, we can expect that in linear analysis the 3D MITC element will also be stable and will not lock, because a minimum number of truss elements is used. The use of the truss structure to idealize the solid element is similar to how the classical transverse shear assumption was developed by Dvorkin and Bathe for 4-node shell elements, notably for the MITC4 shell element [15]. Here the 4-node shell element transverse shear behavior was idealized by the behavior of four 2-node isoparametric beam elements located along the edges of the shell element, with each beam assuming a constant transverse shear strain [1,15].

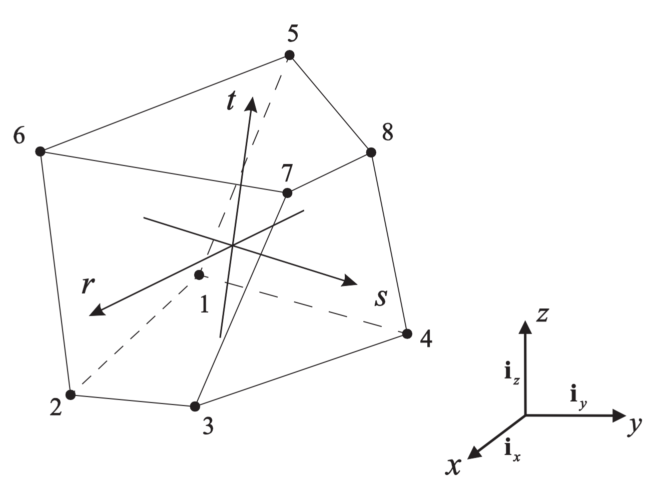

Fig. 1. A standard 8-node hexahedral 3D solid element.

We place a tying location at the center of each truss, and interpolate the assumed strain components according to these locations, see Fig. 2(c). The normal strains (${}_0\tilde{e}_{rr}$, ${}_0\tilde{e}_{ss}$ and ${}_0\tilde{e}_{tt}$) are interpolated bilinearly over the planes defined by their respective tying locations and the shear strains (${}_0\tilde{e}_{rs}$, ${}_0\tilde{e}_{st}$ and ${}_0\tilde{e}_{tr}$) are interpolated linearly between the tying points.

While the resulting assumed strain field yields stability in linear analysis, there is an instability that can arise in nonlinear analysis. The phenomenon has been widely observed for enhanced assumed strain elements when initially regular meshes undergo compression [7–14]. Indeed, for the incompatible modes elements, spurious bending deformations or hour-glass modes are seen at a critical state even in small strains [8]. For an 8-node hexahedral element, possible 2D and 3D hour-glass modes are depicted in Fig. 3(a) and (b), respectively. This behavior occurs if an 8-node element has the ability to express pure bending deformations and the surrounding elements cause mixed behavior of bending and compression. The behavior is possible for the incompatible modes element.

The mechanism of this nonlinear instability was studied by Sussman and Bathe [8], where it was found that a spurious bending deformation occurs when a critical compressive strain state is reached. For the two different kinds of hour-glass modes we treat the potential instabilities separately. We suppress the accumulation of a 3D hour-glass mode by using the incremental displacements to calculate the constant compressive strain. Further, we suppress the 2D hour-glass modes and their coupling to the 3D hour-glass mode by interpolating an additional stabilizing shear strain term bilinearly on the respective planes defined by the mid-points on the edges, see Fig. 3(c).

Incorporating these ideas, the assumed strain field is proposed as

$$
\begin{aligned}
{}_0\tilde{e}_{rr} &= A_{rr}^0 + A_{rr}^1 s + A_{rr}^2 t + A_{rr}^3 st,\\
{}_0\tilde{e}_{ss} &= A_{ss}^0 + A_{ss}^1 t + A_{ss}^2 r + A_{ss}^3 tr,\\
{}_0\tilde{e}_{tt} &= A_{tt}^0 + A_{tt}^1 r + A_{tt}^2 s + A_{tt}^3 rs,\\
{}_0\tilde{e}_{rs} &= A_{rs}^0 + A_{rs}^1 t + S_{rs}^1 r + S_{rs}^2 s + S_{rs}^3 rs,\\
{}_0\tilde{e}_{st} &= A_{st}^0 + A_{st}^1 r + S_{st}^1 s + S_{st}^2 t + S_{st}^3 st,\\
{}_0\tilde{e}_{tr} &= A_{tr}^0 + A_{tr}^1 s + S_{tr}^1 t + S_{tr}^2 r + S_{tr}^3 tr,
\end{aligned}
\tag{1}
$$

in which the A_{ij}^k and S_{ij}^k are the unknown strain coefficients. The constants A_{ij}^k ($k = 0,\ 1,\ 2,\ 3$) and corresponding interpolations allow overall stability in linear and nonlinear analyses. The constants S_{ij}^k ($k = 1,\ 2,\ 3$) are designed to automatically suppress 2D hour-glass deformations in nonlinear solutions and are significant only when compressive strain has been accumulated.

3. Formulation of 3D-MITC8 element

We use the left-superscript t to denote the current configuration (or 'time') of the element. We employ the total Lagrangian formulation with the reference configuration at time *0* indicated by the left subscript *0*.

The geometry and displacement of the standard 8-node hexahedral 3D solid element is interpolated by [1]

$$
\begin{aligned}
{}^t\mathbf{x} &= \sum_{i=1}^{8} h_i(r,s,t)\,{}^t\mathbf{x}_i\\
&= {}^t x\mathbf{i}_x + {}^t y\mathbf{i}_y + {}^t z\mathbf{i}_z\\
&= [{}^t x \quad {}^t y \quad {}^t z]^T,
\end{aligned}
$$

with ${}^t\mathbf{x}_i = [{}^t x_i \quad {}^t y_i \quad {}^t z_i]^T$,