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Effective elastic modulus of heterogeneous peristaic bar of periodic structure

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ABSTRACT

The basic feature of the peridynamic model (introduced by Silling: J. Mech. Phys. Solids, 2000; 48: 175–209) considered is a continuum description of a material behavior as the integrated nonlocal force interactions between infinitesimal material points. A heterogeneous bar of periodic structure of constituents with the peristatic mechanical properties is analysed. One introduces the new volumetric periodic boundary conditions (PBCs) at the interaction boundary of a representative unit cell (UC) whose local limit implies the known locally elastic PBCs. The discretization of the equilibrium equation for peristatic composite materials (CMs) acts as a macro-to-micro transition of the deformation-driven type, where the overall deformation is controlled. Determination of the microstructural displacements in an accompany with the volumetric PBC allows one to estimate the peristatic traction at the geometrical UC's boundary which is exploited for estimation of the macroscopic stresses with subsequent evaluation of the effective moduli. Introduction of the volumetric PBCs opens the opportunities for systematic generalization of the classical computational homogenization approaches for CMs with the local constitutive laws for the different dimensions and physical phenomena to their peristatic counterparts. In particular, a convergence of effective modulus estimations is demonstrated for both the peristatic composite bar and locally elastic bar.

1. Introduction

Peridynamics is a nonlocal theory introduced by Silling [1] (see also [2,3] and references herein) in solid mechanics and generalized in so-called state-based formulation by Silling et al. [4]. The effectiveness of peridynamic models has already been demonstrated in several sophisticated applications, including damage accumulation, the fracture and failure of composites of deterministic structure, crack instability, the fracture of polycrystals, phase transition, diffusion, and nanofiber networks (see, e.g., [5–7] and references herein). Generally in peridynamics, the state-based approach permits the response of a material at a point to depend collectively on the deformation of all bonds connected to the point within its finite radius horizon (Silling et al. [4], Silling and Lehoucq [7]) via a response function that completely describes the interaction. It means that the forces between two peridynamic nodes depend also on deformations of other bonds surrounding these nodes within the horizons. The horizon can encompass discontinuities or different materials. A simplified version derived from this approach is the so-called bond-based approach, in which interactions only occur between pairs of material points within a horizon. As is well known, a direct consequence of this assumption is that the Poissons ratio for isotropic linear materials is fixed at = 1/4 in three dimensions or = 1/3 in two dimensions (plane stress, see [1,8]). The major advantages of the state-based approach include a material response depending on collective quantities (like volume change or shear angle), which allows constitutive models from the conventional theory of solid mechanics to be incorporated directly within the peridynamic approach (see, for example [4,9]). However, this paper will use the bond-based approach as it is most suitable to the chosen implementation. The term peristatics is used analogously to Mikata [10] to differentiate the static problems considered in the current paper from the dynamic problems.

The mentioned achievements of peridynamics were mostly performed for either the initially homogeneous materials or the deterministic structures. However, estimation of macroscopic effective response of heterogeneous media (with either random or periodic structures) in an averaged (or homogenized) meaning in terms of the mechanical and geometrical properties of constituents is not as well developed. Background of random structure peristatic composites was developed by Buryachenko [11,12] who also presented some numerical results for 1D case (see [13,14]). The research in homogenization of peridynamic periodic structure peristatic composite materials (CMs) is less well developed and just a few discussions in this area are published. So, Alali and Lipton [15] constructed two-scale solution expansion (with neither numerical

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results nor definition of the effective moduli) splitting into a microscopic component tracking the dynamics at the length scale of the heterogeneities and a macroscopic component tracking the volume averaged (homogenized) dynamics. By considering the micromechanics of a layered composite under uniaxial stress, Silling [16] demonstrated that nonuniformity of the macroscopic strain field leads to nonlocality in a homogenized model. The peristatic counterpart of the computational homogenization in local elasticity (see e.g. [17–19] and references therein) is started by Madenci et al. [20] who proposed the peridynamic unit cell model for prediction of the effective properties by the use of the classical periodic boundary conditions (PBC). The most important features of the computational homogenization is a description of the periodic boundary conditions at the unit cell. The current work is dedicated to systematic generalization of classical locally elastic version of the computational homogenization to their peristatic counterpart. We refer to the boundary of the UC separating the adjoining UCs one from another as the geometrical boundary of the UC. For the peristatic linear bond-based model being considered and utilizing the integral governing equation, the material points in the vicinity of the geometric boundary of the UC directly interact within the horizon. It defines a volumetric region (called an interaction boundary of the UC) surrounding the geometric boundary and limited by the surfaces at the horizon distance from the geometrical boundary of the UC. Proposal of new *volumetric periodic boundary conditions (PBC)* opens the great opportunities for exploration of peristatic CM of periodic structure. We focus on deformation driven procedure formulated at the macroscopic level as follows: given a macroscopic strain determine the macroscopic stress and the constitutive tangent, based on the response of the underlying microstructure. For clarity, one considers the examples for an infinite 1D peristatic bar of periodic structure when a classical locally elastic counterpart of the corresponding problem is exactly solved. Analysis of the simplified 1D structure makes it possible to focuss our attention to direct use of a large body of both the analytical and numerical results obtained for 1D homogeneous and inhomogeneous peridynamic bar of deterministic structure (see, e.g., Silling et al. [21], Silling and Askari [22], and also [10,23–26]).

The paper is organized as follows. In Section 2 we give a short introduction into the 1D peristatic theory of solids as well as a decomposition scheme for the material and field parameters of heterogeneous peristatic bar. In Section 3 the known exact solutions for locally elastic heterogeneous bar of both statistically homogeneous and periodic structures are summarised in the form adopted for subsequent comparison with the corresponding solutions for the peristatic heterogeneous bar. In Section 4 the problem for one inclusion inside a homogeneous bar is considered at the volumetric displacement loading and forth loading; the proposed quadrature solution forms are adopted for a straightforward generalization in Section 5 to the peristatic bar of periodic structure. In Section 5 the new volumetric periodic boundary conditions are defined and the peristatic problem for the displacement and stresses in the periodic structure bar is solved by both the direct and decomposition quadrature approaches. The representations for the effective properties are obtained in Section 6. The numerical results are presented in Section 7 where one also demonstrates a convergence of effective modulus estimations obtained for the peristatic composite bar to the corresponding exact effective moduli evaluated for the local elastic theory.

2. Preliminaries

2.1. Basic equations of peristatics

In this section, we first summarize the linear peristatic 1D theory (see the references in Introduction) for an infinitely long bar of a constant cross section $A = 1$, assume that the bar is parallel to the $x_1 \equiv x$ axis. We reproduce (see for details Silling et al. [21] the constitutive law for a peristatic bar directly in the one-dimensional setting, omitting the calculations requiring the cross section:

$$\mathcal{L} * u(x) + b(x) = 0, \quad \mathcal{L} * u(x) := \int_{-\infty}^{\infty} C(x,\hat{x})[u(\hat{x}) - u(x)]d\hat{x}, \qquad (2.1)$$

where u is the displacement field, b is a prescribed external force density field, and C is a stiffness distribution density or micromodulus function. The body force density function $b(x)$ is assumed to be self-equilibrated

$$\int_{-\infty}^{\infty} b(x)dx = 0 \qquad (2.2)$$

and vanished outside some loading region: $b(x) = 0$ for $|x| > a^\delta$. For consistency with Newton′s third law, the micromodulus function C for the homogeneous materials must be even ($\xi = x - \hat{x}$):

$$C(\xi) = C(-\xi) \quad \text{for} -\infty < \xi < \infty. \qquad (2.3)$$

It is assumed that $C(\xi)$ has a compact support, i.e. the material has a "horizon", when there is no interaction between particles that are more than some finite distance l_c apart, then $C(\xi) \equiv 0$ for all $|\xi| > l_c$. Thus, the integration domain $R = (-\infty, \infty)$ in Eq. (2.1) can be limited by a neighborhood $\mathcal{H}_x(\hat{x}) = \{\hat{x} : \ |\hat{x} - x| < l_c\}$ of the point x. For example for the micromodulus functions with the step-function and triangular profiles

$$C(\xi) = \begin{cases} C, & \text{for } |\xi| < l_c, \\ 0, & \text{for } |\xi| > l_c. \end{cases} \quad C(\xi) = \begin{cases} C(1 - |\xi|/l_c), & \text{for } |\xi| < l_c, \\ 0, & \text{for } |\xi| > l_c, \end{cases} \qquad (2.4)$$

respectively.

For two phase bar, the domain R contains a homogeneous matrix $v^{(0)}$ and a periodic (or statistically homogeneous) field $X = (v_i)$ of identical inclusions $v_i \subset v^{(1)}$ with indicator functions V_i ($v^{(0)} \cup v^{(1)} = R$, $v^{(0)} \cap v^{(1)} = \varnothing$) and length $2a$, e.g. $v_0 = \{x : \ |x| < a\}$. For statistically homogeneous field X, we consider a dilute approximation when interaction of inclusions $v_i \subset v^{(1)}$ are absent, and the peridynamic horizon l_c is chosen to be smaller than the spacing separating the inclusions. For the periodic field X, the mentioned restriction is eliminated. For any two points x and $\hat{x}$ in $R, C(\xi) = C(x,\hat{x})$ ($\xi = x - \hat{x}$) is given by the formula ($v_i \subset v^{(1)}, i = 0, 1, 2, \ldots$)

$$C(x,\hat{x}) = \begin{cases} C^{(1)}(x,\hat{x}), & \text{for } x,\hat{x} \in v_i, \\ C^{(0)}(x,\hat{x}), & \text{for } x,\hat{x} \in v^{(0)}, \\ C^{i}(x,\hat{x}), & \text{for } x \in v_i, \hat{x} \in v^{(0)} \text{or} \quad x \in v^{(0)}, \hat{x} \in v_i, \\ 0, & \text{for } |x - \hat{x}| > l_c, \end{cases} \qquad (2.5)$$

which can also be presented in the form

$$C(x,\hat{x}) = C^{(1)}(x,\hat{x})V^{(1)}(x)V^{(1)}(\hat{x}) + C^{(0)}(x,\hat{x})V^{(0)}(x)V^{(0)}(\hat{x}) + C^{i}(x,\hat{x})[V^{(1)}(x)V^{(0)}(\hat{x}) + V^{(0)}(x)V^{(1)}(\hat{x})], \qquad (2.6)$$

where $V^{(k)}(x)$ is an indicator function of $v^{(k)}$ equals 1 at $x \in v^{(k)}$ and 0 otherwise ($k = 0, 1$). The material parameters $C^{(1)}$ and $C^{(0)}$ are intrinsic to each phase and can be determined through the experiments. Bonds connecting particles in the different materials are characterized by micromodulus C^i, which can be chosen such that $C^{(1)}(x,\hat{x}) \geqslant C^{i}(x,\hat{x}) \geqslant C^{(0)}(x,\hat{x})$, or

$$C^{i}(x,\hat{x}) = (C^{(0)}(x,\hat{x}) + C^{(1)}(x,\hat{x}))/2, \qquad (2.7)$$

$$C^{i}(x,\hat{x}) = \min[C^{(0)}(x,\hat{x}), C^{(1)}(x,\hat{x})], \qquad (2.8)$$