Computers and Structures 201 (2018) 37–57

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Microscale analysis of heterogeneous ductile materials with nonlocal damage models of integral type

F.J.P. Reis, I.A. Rodrigues Lopes, F.M. Andrade Pires *, F.X.C. Andrade

DEMec - Department of Mechanical Engineering, Faculty of Porto, University of Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal

ARTICLE INFO

Article history:
Received 22 June 2017
Accepted 24 February 2018

Keywords:
Ductile materials
Heterogeneous microstructure
Homogenization
Nonlocal damage

ABSTRACT

A computational scheme for the analysis of damage localization in heterogeneous ductile materials in the case of non-separated scales is presented. The consistent linearization of nonlocal damage models of integral type at finite strains is addressed and the influence of nonlocal interactions on the homogenized material response is investigated. The constituents and phases of the material at the microstructural representative volume element (RVE) level are modeled with a nonlocal elasto-plastic isotropic damage model. The numerical integration of the constitutive equations within a nonlinear homogenization problem is described in detail. The scheme is applied to the simulation of ductile damage and the influence of the nonlocal averaging procedure, which can be evaluated at different configurations and include nonlocal interactions between different phases, is analysed. The quadratic rate of convergence of the Newton-Raphson iterative procedure is demonstrated and the capability to alleviate the pathological mesh dependence is illustrated through microstructural examples.

1. Introduction

The prediction of the mechanical behavior of heterogeneous ductile materials undergoing finite strains and subjected to arbitrary conditions, possibly leading to damage and failure, is extremely important for a wide variety of applications including structural engineering, forming operations, collision of solids, among others. Therefore, considerable efforts have been made to understand the deformation behavior of these materials and to develop constitutive models which are able to capture the experimentally observed response. Several macroscopic constitutive models, based on either micro-mechanical information [1–4] or phenomenological assumptions [5,6], have been proposed. They usually introduce an appropriate set of internal variables, whose growth is governed by evolution laws, to represent the underlying dissipative mechanisms (such as plasticity or damage). Although these constituive models have proved their merit over a wide range of applications, they do not explicitly account for the microstructural morphology or the interaction between different constituents. This can be particularly important for materials with complex microstructures undergoing complex loadings.

An alternative strategy consists in using a confined model of the microstructure, usually known as unit cell or representative volume element (RVE), which incorporates all statistically relevant microstructural features, to capture the material constitutive response [7–9]. The so-called micromechanical approach requires the solution of a boundary value problem of the RVE based on the knowledge of the macroscopic deformation tensors and internal variables, to obtain the macro stress from a homogenization procedure. The behavior of the different constituents and phases at the RVE level is typically modelled by local continuum constitutive models following the formalism of thermodynamics with internal variables.

The evolution of the dissipative mechanisms at the microscale may lead to the onset of macroscopic material failure. Therefore, several multiple scale approaches have been presented in the last years to link the evolution of the microscopic fields with macroscopic failure [10–13]. Here, focus is placed on homogenization-based multi-scale constituive models, also known as FE^2, where the macroscopic stress and strain tensors are defined as volume averages of their microscopic counterparts over a Representative Volume Element (RVE) of material [14–17]. These constituive models are based on the nested solution of two coupled problems, one at a macroscale and other at the micro-scale.

The description of the deformation behaviour of the material close to rupture, for all constituive approaches, is typically performed by the introduction of damage variable(s) that quantify the loss of load carrying capacity due to the presence and evolution of defects at finer scales. Unfortunately, the use of constitutive

* Corresponding author.
E-mail address: fpires@fe.up.pt (F.M. Andrade Pires).

https://doi.org/10.1016/j.compstruc.2018.02.013
0045-7949/

models inducing softening within a local continuum framework can lead to the loss of ellipticity of the governing equilibrium equations after the onset of strain-softening. Consequently, the boundary value problem becomes ill-posed and the associated finite element solution becomes dependent of the spatial discretization [18–22]. In order to minimize this mesh dependence pathology, several *non-local* approaches have been developed over the last years. The main idea behind these formulations is to introduce an intrinsic length into the standard constitutive models in order to properly define the localization zone. Two different approaches, within the context of non-local formulations, can be found in the literature: gradient-type and integral-type theories [23–30].

In the gradient-type nonlocal strategy, an additional equation is added to the structural problem - *the diffusivity equation* - and the non-local variable is considered as an additional unknown of the global problem, which requires additional constraints to have a well-posed problem. Therefore, the modelling of different materials within the same structure can be very challenging due to the need to introduce additional boundary constraints and the increased size of the problem. On the other hand, this strategy keeps the material constitutive level unchanged, which can be an advantage. In contrast, the non-local approach of integral-type follows a distinct philosophy. The non-locality effect is embedded at the material level instead of the structural domain. Hence, no modifications are required at the global problem. However, whenever a new constitutive model is developed, it is necessary to redefine the constitutive equations to comply with the non-local integral-type rules. Over the last decades, while the non-local model of gradient-type has been the focus of many studies and has been significantly developed [23–26], the non-local method of integral-type has received less attention. Recently, this approach has been extended for elasto-plastic materials [27–29] and to finite strains [30], where the derivation of a closed form for the material tangent modulus has widened the range of applicability of non-local models of integral-type.

In homogenization-based multi-scale constitutive models, the lack of objectivity of the solution may arise from either macroscale or microscale domains. This has triggered an interesting discussion about the applicability of first-order homogenization schemes and the existance of a RVE [31]. Second-order multiscale models [32], by naturally introducing size effects into the homogenization procedure, are capable of capturing moderate localization at the macroscale [33]. Other techniques have also been proposed by several authors. Gitman et al. [34] introduced a coupled-volume strategy for quasi-brittle materials, where the RVE size is directly related to the macro-elements size, which allows the elimination of both macro-level mesh and meso-level cell size dependence. Nguyen et al. [35] have shown that with an alternative homogenization procedure, where the averaging procedure is carried out on the damaged zone, a RVE can be defined for concrete-like materials undergoing softening. A multiscale failure model for concrete was later proposed [36]. In these works, a gradient enhanced damage model is used at the microscale to overcome mesh dependency. Fish et al. [37] have proposed a multiscale model for heterogeneous materials (at small strains) that combines reduced-order homogenization with an integral nonlocal formulation.

The modelling of damage localization and the transition between different scales is still a challenging task [38–42] since when localization occurs, the separation of scales is intrinsically violated. Nevertheless, the need to regularize the solution at the meso-level in the presence of strain softening is unavoidable in order to obtain meaningful results. Therefore, several strategies have been proposed [31,34,36] to avoid the loss of ellipticity in constitutive models at the onset of strain softening. Nevertheless, these approaches are based on gradient-type non-local formulations and have been mainly applied for quasi-brittle materials. Therefore, the main goal of this contribution is to present a computational scheme for the analysis of damage localization in ductile heterogeneous materials at finite strains with non-separated scales. This strategy is based on an elasto-plastic integral-type nonlocal formulation where the damage evolution is driven by the level of equivalent plastic strain and each nonlocal variable is computed using a different nonlocal kernel function. Particular attention is given to the numerical implementation of the framework within an implicit quasi-static finite element scheme and emphasis is given to the consistent linearization of the problem at finite strains. The influence of nonlocal interactions on the homogenized material response is investigated and the ability of the nonlocal theory of integral type to minimize the mesh-dependence pathology at the meso-level equilibrium problem is assessed for several RVE sizes and distinct realizations, under different boundary conditions.

The work is organized as follows. In Sections 2 and 3, we briefly revise some general concepts and definitions of the multiplicative hyperelasto-plasticity framework and the micromechanical homogenization strategy that will be used in this contribution. Then, in Section 4, the main ingredients of non-local methods of integral type are introduced. For more details see references [23,20,43,29]. A non-local framework of integral type is proposed for heterogeneous media modelled by means of isotropic explicit damage models, in Section 5. The numerical treatment of the integration algorithm and the derivation of the consistent tangent operator for a non-local explicit damage model, within a nonlinear homogenization problem, is presented in Section 6. In Section 7 the proposed numerical framework is assessed by means of two numerical examples. Finally, a study of the influence of the RVE size and boundary conditions on the non-local damage is conducted in Section 8. The present contribution finishes with some remarks and conclusions in Section 9.

2. General kinematics of hyperelastic-based multiplicative plasticity

In this section, a brief review of some important definitions that will be employed in forthcoming sections is presented. A more detailed discussion of the multiplicative hyperlasto-plasticity framework can be found in [44–48]. The deformation gradient, $\boldsymbol{F}$, can be decomposed in the product

$$\boldsymbol{F} = \boldsymbol{F}^e \boldsymbol{F}^p, \tag{1}$$

where $\boldsymbol{F}^e$ and $\boldsymbol{F}^p$ are, respectively, the elastic and plastic deformation gradients. The multiplicative split of $\boldsymbol{F}$ assumes the existence of a local unstressed intermediate configuration that corresponds to a deformed configuration which has been elastically unloaded.

The polar decomposition can be straightforwardly employed within the multiplicative framework and we obtain the following useful relations:

$$\boldsymbol{F}^e = \boldsymbol{R}^e \boldsymbol{U}^e = \boldsymbol{V}^e \boldsymbol{R}^e, \tag{2}$$

$$\boldsymbol{F}^p = \boldsymbol{R}^p \boldsymbol{U}^p = \boldsymbol{V}^p \boldsymbol{R}^p, \tag{3}$$

where $\boldsymbol{R}^e$, $\boldsymbol{U}^e$ and $\boldsymbol{V}^e$ are the elastic rotation, right and left stretch tensors, respectively. Likewise, $\boldsymbol{R}^p$, $\boldsymbol{U}^p$ and $\boldsymbol{V}^p$ are named the plastic rotation, right and left stretch tensors.

The velocity gradient, which can be defined as

$$\boldsymbol{L} \equiv \dot{\boldsymbol{F}} \boldsymbol{F}^{-1}, \tag{4}$$

may also be expressed in terms of $\boldsymbol{F}^e$ and $\boldsymbol{F}^p$:

$$\boldsymbol{L} = \boldsymbol{L}^e + \boldsymbol{F}^e \boldsymbol{L}^p (\boldsymbol{F}^e)^{-1} \tag{5}$$