Computers and Structures 200 (2018) 53–67

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

A linear relaxation model for shape optimization of constrained contact force problem

Weihong Zhang *, Cao Niu

State IJR Center of Aerospace Design and Additive Manufacturing, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China

ARTICLE INFO

Article history:
Received 4 August 2017
Accepted 12 February 2018
Available online 2 March 2018

Keywords:
Contact shape optimization
Contact sensitivity analysis
Relaxation model
Leakproofness
Aero-engine structure design

ABSTRACT

This paper is devoted to shape optimization of constrained contact force problem. A linear relaxation model of contact force-clearance relation is developed. It is shown that such relaxation is essential to avoid zero sensitivity values of zero contact force with respect to design variables and to drive the satisfaction of contact force constraint at the specific contact region. In this work, both frictionless and frictional contact optimization problems are investigated by means of a gradient-based optimization algorithm. The optimization procedure including sensitivity analysis with relaxation model is implemented outside the structural contact analysis software to deal with non-smooth contact problems. The procedure is firstly validated by simple examples with analytical solutions and is finally applied to the design of an assembled aero-engine structure for leakproofness.

1. Introduction

It is common to design a mechanical structure as an assembly of components for the realization of desired functionalities. Contacts thus become natural and inevitable in practical applications related to connection, fixation, sealing, and load/motion transmission, etc. It is found that contact forces/pressures along contact surfaces essentially influence the performances of the assembled structure, e.g., torque transmission capability of interference fits, leakproofness of contact seals and service life due to wear and fatigue. Generally, contact forces are very sensitive to the shape of contact surface, even a slight shape variation of contact surface may result in great changes of the contact forces and their distribution. It is therefore of great importance to develop the so-called contact optimization techniques for the achievement of an elaborate design of the contact surface shapes.

A literature survey indicates that studies related to contact optimization have ranged from linear elastic frictionless [1–4] to large deformation frictional [5–7] contact problems until now. Emerging methods such as level set method [8–10] and phase field method [11] were also extended to contact optimization. However, relevant works are rather limited in comparison with the optimization of a single mechanical part despite the ubiquitousness and importance of the former in practice [12]. As the attainment of a uniform contact pressure distribution has the great advantage for wear reduction or prolongation of the fatigue life [2,13–19], constraining the contact force with an upper bound was studied by Klarbring and Rönnqvist [20] in truss weight minimization. In fact, even a zero contact force is feasible to easily satisfy such upper bounded constraint although a zero contact force is always accompanied by zero sensitivities with respect to (w.r.t.) design variables [20–22]. Besides, interference fit designs with lower bounded constraints to the total contact force were treated in [22–24] to ensure a sufficient torque transmission capability.

However, the contact pressure at each point along the contact surface should be individually constrained by a certain positive value as a lower bound [24] in contact seal design. This kind of constraint, also referred to as the leakproofness constraint, indeed reflects the design requirement of contact seals such that the so-called percolation limit [25] is reached for a reliable contact seal. The main challenge is how to avoid possible zero normal contact forces that are infeasible for the lower bounded constraint. In other words, since a zero contact force with zero sensitivities is unable to guide the optimization process for convergence, the gradient-based algorithm will fail to find out a feasible search direction in the presence of lower bounded constraints of contact pressures at the specific contact region.

To circumvent this difficulty, it is intuitive that a decrease of the clearance related to a zero contact force has the tendency of coming into active contact for the corresponding non-contact node pair, although the real contact force may still remain zero. That is to say, the contact clearance and contact force can be considered to vary in opposite tendency. With this idea in mind, a relaxation

* Corresponding author.
E-mail address: zhangwh@nwpu.edu.cn (W. Zhang).

https://doi.org/10.1016/j.compstruc.2018.02.005
0045-7949/

model is proposed to quantify one such tendency between the contact force and clearance. This makes it possible to guide the gradient-based contact optimization process for the convergence and for the satisfaction of the lower bounded contact pressure constraints even though they are infeasible at the beginning.

In this work, contact shape optimization is carried out based on a node-to-node (NTN) contact scheme of finite element (FE) model within the regime of linear elasticity. The commercial software ANSYS is employed as the contact problem solver. Without any modification of its kernel, the relaxation model is implemented as a post-processing scheme outside the FE code to handle the lower bounded constraint of contact force.

This paper is organized as follows. In Section 2, the FE formulation of NTN contact discretization is briefly presented. In Section 3, sensitivity analysis formulations are derived for both frictionless and frictional contact problems in consideration of the relaxation model. In Section 4, a variety of examples from 1D elastic bar-rigid wall contact problem, classical cylinder-cylinder Hertz contact problem with analytical solutions to shape optimization design of an aero-engine low-pressure turbine (LPT) assembly structure are tested to validate sensitivity analysis formulations and optimization procedure. Finally, conclusions are drawn out in Section 5.

2. FE formulation of contact problem with NTN contact discretization

Without loss of generality, Fig. 1 depicts two planar elastic bodies related to domains Ω^{I} and Ω^{II} in contact. Γ_c^{I} and Γ_c^{II} denote the potential contact surfaces. Γ_c denotes the real contact surface that is usually unknown until the contact problem is solved. Fig. 2 depicts the NTN contact discretization with 2D quadrilateral elements. At the potential contact surfaces, conformal meshes are defined with nodes of a body aligned with those of the counterpart. Note that finer meshes at the potential contact surfaces are generally needed because the FE representation of the geometric boundary is not smooth [26]. $\mathbf{x}_i^{\text{I}}$ and $\mathbf{x}_i^{\text{II}}$ denote the coordinate vectors representing a contact node pair indexed by i ($i = 1, 2, \ldots, n_{\text{p}}$) along the potential contact surfaces in the undeformed configuration. n_{p} denotes the number of potential contact node pairs. Although the normal unit vector $\mathbf{n}_i$ is depicted as pointing from node $\mathbf{x}_i^{\text{I}}$ to node $\mathbf{x}_i^{\text{II}}$, it can also be defined in the opposite direction, in which case the tangential unit vector $\mathbf{t}_i$ should reverse its direction as well.

The normal contact gap function related to contact node pair i is defined as

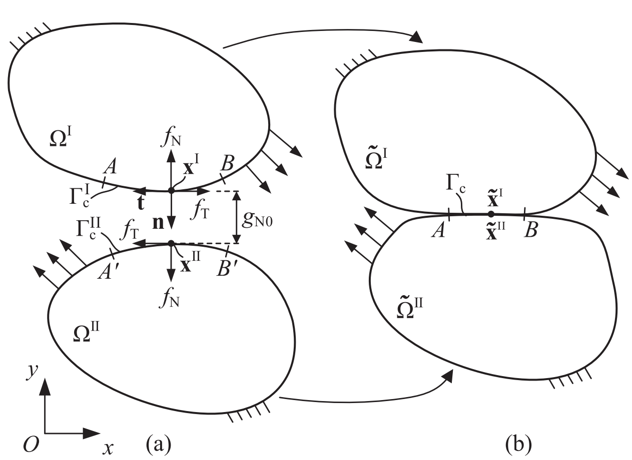

Fig. 1. Contact between two bodies. (a) Undeformed configuration, (b) deformed configuration.

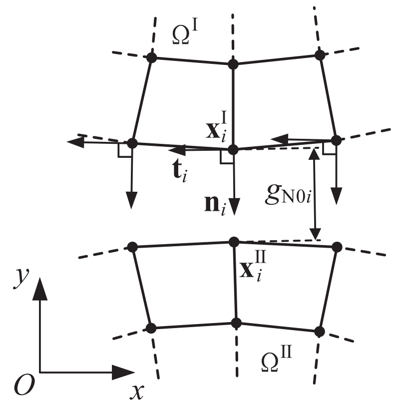

Fig. 2. Node-to-node contact discretization at undeformed configuration.

$$
\begin{aligned}
g_{\text{N}i} &= \left(\tilde{\mathbf{x}}_i^{\text{II}} - \tilde{\mathbf{x}}_i^{\text{I}}\right) \bullet \mathbf{n}_i \\
&= \mathbf{C}_{\text{N}i}^{\text{T}}(\mathbf{u}_i + \mathbf{x}_i) = \hat{\mathbf{C}}_{\text{N}i}^{\text{T}}(\mathbf{u} + \mathbf{x})
\end{aligned} \tag{1}
$$

with

$$
\mathbf{u}_i = \begin{Bmatrix} \mathbf{u}_i^{\text{I}} \\ \mathbf{u}_i^{\text{II}} \end{Bmatrix} = \begin{bmatrix} u_{xi}^{\text{I}} & u_{yi}^{\text{I}} & u_{xi}^{\text{II}} & u_{yi}^{\text{II}} \end{bmatrix}^{\text{T}} \tag{2}
$$

$$
\mathbf{x}_i = \begin{Bmatrix} \mathbf{x}_i^{\text{I}} \\ \mathbf{x}_i^{\text{II}} \end{Bmatrix} = \begin{bmatrix} x_i^{\text{I}} & y_i^{\text{I}} & x_i^{\text{II}} & y_i^{\text{II}} \end{bmatrix}^{\text{T}} \tag{3}
$$

$$
\mathbf{C}_{\text{N}i} = \begin{Bmatrix} -\mathbf{n}_i \\ \mathbf{n}_i \end{Bmatrix} = \begin{bmatrix} -n_{xi} & -n_{yi} & n_{xi} & n_{yi} \end{bmatrix}^{\text{T}} \tag{4}
$$

in which $\tilde{\mathbf{x}}_i^{\text{I}}$ and $\tilde{\mathbf{x}}_i^{\text{II}}$ are coordinate vectors in the deformed configuration. $\mathbf{u}_i^{\text{I}}$ and $\mathbf{u}_i^{\text{II}}$ denote corresponding displacement vectors. $\mathbf{u} = \begin{Bmatrix} \mathbf{u}^{\text{I}} \\ \mathbf{u}^{\text{II}} \end{Bmatrix}$, $\mathbf{x} = \begin{Bmatrix} \mathbf{x}^{\text{I}} \\ \mathbf{x}^{\text{II}} \end{Bmatrix}$ collects all the nodal displacement vectors and nodal coordinates of the two discretized bodies, respectively. $\mathbf{T}_i \in \mathbb{R}^{4 \times n_{\text{d}}}$ is a coefficient matrix to extract $\mathbf{u}_i$ from $\mathbf{u}$, i.e., $\mathbf{u}_i = \mathbf{T}_i\mathbf{u}$. $\hat{\mathbf{C}}_{\text{N}i} = \mathbf{T}_i^{\text{T}}\mathbf{C}_{\text{N}i}$ refers to the normal kinematic transformation vector of contact node pair i. $n_{\text{d}} = n_{\text{d}}^{\text{I}} + n_{\text{d}}^{\text{II}}$ is the total number of DOFs of the whole contact system after FE discretization. n_{d}^{I} and n_{d}^{II} denote the numbers of DOFs of bodies I and II, respectively.

It should be mentioned that the second item in Eq. (1) is related to the initial gap

$$
g_{\text{N}0i} = \hat{\mathbf{C}}_{\text{N}i}^{\text{T}}\mathbf{x} \tag{5}
$$

Notice that a positive value of initial gap ($g_{\text{N}0i} > 0$) indicates the existence of initial clearance, while a negative value ($g_{\text{N}0i} < 0$) implies the existence of initial interference.

Both frictionless and frictional contact problems are considered below.

2.1. Frictionless contact problem

Theoretically, only normal contact behaviour along the contact surface needs to be considered for a frictionless contact problem. According to [27], the normal contact behaviour is governed by the well-known Hertz–Signorini–Moreau conditions. At contact node pair i, we have

$$
g_{\text{N}i} \geqslant 0, \; f_{\text{N}i} \leqslant 0, \; f_{\text{N}i}g_{\text{N}i} = 0 \tag{6}
$$

As illustrated in Fig. 3(a), contact force vanishes in the case of non-contact ($g_{\text{N}i} > 0, f_{\text{N}i} = 0$), while contact occurs with a zero contact gap and a compressive contact force ($g_{\text{N}i} = 0, f_{\text{N}i} \leq 0$). It can be seen that Eq. (6) refers to a multivalued and non-smooth relation due to the singular point at the origin ($g_{\text{N}i} = 0, f_{\text{N}i} = 0$).