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An explicit algorithm for geometrically nonlinear transient analysis of spatial beams using a corotational total Lagrangian finite element formulation

Chu Chang Huang [a], Fumio Fujii [b], Kuo Mo Hsiao [a,*]

[a] Department of Mechanical Engineering, National Chiao Tung University, Hsinchu, Taiwan
[b] Department of Mechanical Engineering, Gifu University (Emeritus), Gifu, Japan

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ABSTRACT

An explicit method for nonlinear transient dynamic analysis of spatial beams with finite rotations using a corotational total Lagrangian finite element formulation is presented. The kinematics of the beam element is described in the current element coordinate system constructed in the current configuration of the beam element. The element deformation and inertia nodal forces are derived by the virtual work principle, the d'Alembert principle, and the consistent linearization of the geometrically nonlinear beam theory. A nodal rotation vector is used to represent the finite rotation of a base coordinate system rigidly attached to each node of the discretized structure. A numerical procedure of explicit method is proposed for the solution of the nonlinear equations of motion. The standard central difference method is applied to the incremental displacement vector and the incremental rotation vector, and the time derivatives of displacement vector and rotation vector. The nodal orientations are updated by the incremental nodal rotation vectors. The values of nodal rotation vectors are reset to zero in the current configuration.

In order to assess the efficiency and the accuracy of the proposed method, numerical examples are studied and compared with the results obtained using the implicit method based on the Newmark method.

1. Introduction

The implicit methods based on the Newmark direct integration method have been extensively employed in nonlinear transient dynamic analysis of three-dimensional beam structures undergoing large displacements and finite rotations using the total Lagrangian (TL) formulation [1–13], the corotational total Lagrangian (CR-TL) formulation [14], the corotational (CR) formulation [15–17], and the combination of CR and TL formulation [18]. In [14], a consistent CR-TL formulation is proposed to derive the deformation nodal force vector and the inertia nodal force vector. In [16,17], a consistent CR framework is used to derive the internal force vector and the tangent stiffness matrix, and the inertia force vector and the tangent dynamic matrix. In [18], the CR formulation is used to develop expressions of the deformation force vector and the tangent stiffness matrices, while the inertia force vector and the tangent inertia matrix are derived using the TL formulation. It seems that the deformation forces and the inertia forces may be derived by adopting different formulations.

The CR-TL formulation is an approach blending the TL and CR descriptions [19]. A good description of the CR formulation and its relation to the TL formulation is given by Mattiasson and Samuelsson [20]. In the TL formulation, the reference configuration is the initial undeformed element configuration. The reference configuration used in the CR formulation is a corotated configuration, which is obtained as a rigid body motion of the reference configuration of the TL formulation from the initial to the current (or neighboring) element configuration. The reference configurations of the CR and CR-TL formulation are coincident at the current state. However, the reference configuration of the CR formulation is a moving frame, while that of the CR-TL formulation is an inertial frame which is kept fixed at the current state. Newton's laws could hold in their simplest form in the inertial frame. Thus, it may be easier to derive the inertia forces using the CR-TL formulation than using the CR formulation.

Generally, implicit algorithms are very effective for structural dynamics problems in which the response is predominantly governed by a relatively small number of low frequency modes, while

* Corresponding author.
E-mail address: kmhsiao@mail.nctu.edu.tw (K.M. Hsiao).

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explicit algorithms are efficient for dynamics problems in which the response is more influenced by higher modes. However, the application of the explicit methods in the nonlinear transient dynamic analysis of three-dimensional beams with finite rotations has been rather limited [21–23]. Hence, the object of this paper is to present an explicit integration method based on the central difference method for nonlinear transient analysis of spatial beams with finite rotations using a CR-TL finite element formulation.

In [24], a CR-TL formulation of beam element for the static nonlinear analysis of three-dimensional beam structures is proposed. To treat arbitrarily large rotation of node in space, the orientation of the node is described by a base coordinate system rigidly attached to each node of the discretized structure, and a nodal rotation vector is used to represent the finite rotation of the base coordinate system. The coordinate system associated with the reference configuration of the beam element is chosen to be the current element coordinate system constructed at the current configuration of the element. Therefore, the current element coordinate system is an inertial local coordinate system, not a moving coordinate system. Three rotation parameters are defined in the current element coordinates to determine the orientation of element cross section. The element deformation nodal force is derived by the virtual work principle and the consistent second order linearization of the fully geometrically nonlinear beam theory. This formulation is extended for the nonlinear dynamic analysis of the beam structures in [14]. Because the coordinate system associated with the reference configuration is regarded as an inertial local coordinate system, the first and second time derivatives of the position vector defined in the current element coordinates are the absolute velocity and acceleration. A numerical procedure based on the Newmark direct integration method and the Newton-Raphson method is proposed for the solution of the nonlinear equations of motion. The standard Newmark method with the stiffness matrices and mass matrices of the elementary beam element and bar element was applied to the incremental displacement vector and incremental rotation vector, and their time derivatives. The formulation and numerical procedure were proven to be very effective by numerical examples studied in [14]. Note that the nodal orientations of the discretized structure are updated by the incremental nodal rotation vectors at each time step and iteration, which entails a large storage pool and extra computational operations. Thus, an efficient procedure for orientation update may be indispensible. It can be noted that for incremental rotation vector most of the formulation details have been clarified in [25], both in spatial and material version, including the dynamics. In order to include the nonlinear coupling among the bending, twisting, and stretching deformations, the terms up to the second order of rotation parameters and their spatial derivatives are all retained in the element deformation nodal forces in [14,24]. However, it was reported that the third order term of the twist rate should be retained in the element deformation nodal forces for the geometric nonlinear analysis of thin-walled beams in [26,27]. It was also reported in [26] that the second-order terms of the element deformation nodal force containing the twist angle and slopes the beam element will converge to zero with the decrease of element size, and if these terms are removed from the element nodal forces, the convergence rate of the solution may be increased. It seems that the complexity of the element nodal forces could also be significantly reduced by dropping the second-order terms containing the twist angle and slopes the beam element. Hence, the CR-TL formulation proposed in [14,24,26,27] is adapted and used in this paper. The element deformation and inertia nodal forces are derived by the virtual work principle, the d'Alembert principle, and the consistent second order linearization of the fully geometrically nonlinear beam theory. Since an explicit integration method is used in this paper, the tangential stiffness matrix is not needed and not developed. A numerical procedure of explicit method based on the central difference method is proposed here for the solution of the nonlinear equations of motion. The standard central difference method is applied to the incremental displacement vector and the incremental rotation vector, and the time derivatives of displacement vector and rotation vector. The nodal orientations of the discretized structure are updated by the incremental nodal rotation vectors. Then, the values of nodal rotation vectors are reset to zero in the current configuration. To assess the efficiency and accuracy of the proposed method, numerical examples are studied and compared with results obtained from the implicit numerical procedure based on the Newmark method [14] and the results reported in the literature.

2. Finite element formulation

The kinematics of the beam element and the corotational total Lagrangian finite element formulation proposed in [14,24,26,27] are adapted and employed here.

2.1. Basic assumptions

The following assumptions are made in the derivation of the beam element behavior.

(1) The beam is prismatic and slender, and the Euler-Bernoulli hypothesis is valid.
(2) The cross section of the beam is doubly symmetric.
(3) The unit extension and the twist rate of the centroid axis of the beam element are uniform.
(4) The cross section of the beam element does not deform in its own plane, and strains within this cross section can be neglected.
(5) The out-of-plane warping of the cross section is the product of the twist rate of the beam element and the Saint Venant warping function for a prismatic beam of the same cross section.
(6) The deformation displacements and rotations of the beam element are small.
(7) The strains of the beam element are small.

In conjunction with the CR-TL formulation, the sixth assumption can always be satisfied if the element size is properly chosen. The assumption of small strains may not be required in the CR-TL formulation. However, only the linear elastic material is considered in this study. The yield strains for most engineering materials are small. Thus, the assumption of small strains is used here.

2.2. Coordinate systems

In order to describe the system, we define three sets of right-handed rectangular Cartesian coordinate systems:

(1) A fixed global set of coordinates, X_i^G ($i = 1, 2, 3$) (see Fig. 1); the nodal coordinates, displacements, rotations, velocities, and accelerations, and the equations of motions of the system are defined in these coordinates.
(2) Element cross section coordinates, x_i^S ($i = 1, 2, 3$) (see Fig. 1); a set of element cross section coordinates is associated with each cross section of the beam element. The origin of this coordinate system is rigidly attached to the centroid of the cross section. The x_1^S axis is chosen to coincide with the normal of the unwrapped cross section, and the x_2^S and x_3^S axes are chosen to be the principal directions of the cross section.