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The new paradigm of finite element solutions with overlapping elements in CAD – Computational efficiency of the procedure

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ABSTRACT

We consider the new paradigm of finite element analysis, present an effective overlapping finite element, and study the computational efficiency of the discretization scheme.

The important new ingredient in the formulation of the overlapping element is that, unlike in meshless methods, we only use local polynomial functions in the displacement interpolations. We achieve this property by replacing the Shepard functions by local polynomials. As a consequence, the bandwidth of the resulting stiffness matrix for the overlapping finite element is much reduced when compared with earlier developments.

We study the distortion insensitivity of the new overlapping finite element, the convergence properties and the required computational effort when compared with the use of the traditional 4-node finite element and that element with covers. The results show the overlapping element to be very promising, in particular in the new paradigm of analysis using finite elements in CAD.

1. Introduction

The computational procedures and resources available today make the finite element analysis of complex structures and fluid systems possible. However, due to meshing difficulties, significant experience in building effective meshes is still needed. In practice, much more time may be spent by an engineer to reach an adequate mesh than the time used by the finite element program for the calculation and solution of the governing equations. Indeed, it is well-known that the experience and time required for an analysis impedes the wider use of finite element analysis in the field of computer-aided design (CAD).

To overcome the meshing difficulties, a number of unstructured mesh generation algorithms have been proposed, see e.g. Refs. [1–3]. The goal is to develop a robust and efficient mesh generator that automatically builds high-quality finite element meshes for large and complex geometries. Since the data defining the geometry is frequently not directly usable, the engineer usually has to first clean up the geometry [4]. Then in order to reach an adequate mesh, a considerable amount of engineering time and computational effort may be required [3]. Also, an unstructured mesh generator may perform quite well for simple geometries but may fail in building an adequate mesh for geometrically complex objects.

For these reasons, many meshfree or meshless methods have been proposed, see e.g. Ref. [5]. However, while using a meshfree method the time to establish the discretization is much less and good solution accuracy can be obtained, the method may not be stable unless artificial stability parameters are used, or the required numerical integration may be computationally very expensive, and inherently so, see e.g. Refs. [5–9] and the references therein. These limitations largely restrict the wide use of meshfree methods in engineering practice.

To significantly reduce the meshing effort required in finite element analysis, we proposed a new paradigm of finite element solutions in computer-aided design [10–12]. In the new paradigm, the geometry is obtained from any CAD program or by any other means. Hence the procedure is not limited, for example, to dealing only with geometries represented by NURBS. The geometry is immersed in a Cartesian grid of (usually uniform) cells, the boundary of the geometry is discretized while defects are removed, and cells within the analysis domain are automatically, and with little computational effort, converted to traditional finite elements. Thereafter, overlapping finite elements are inserted to fill-in the empty space and couple with the traditional finite elements. The solution accuracy is good because undistorted traditional finite elements [13,14] and distortion insensitive overlapping finite elements are used [12]. Moreover, since the analysis domain is largely meshed with uniform finite elements, a stress improvement procedure can be used effectively [15].

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This approach is clearly related to schemes using "overlapping grids" in finite difference and control volume solutions of fluid flows, see e.g. Refs. [16–20]. Here finite difference grids are superimposed to cover the complete analysis domain. While our approach for finite element analysis is related to these schemes, it shows much more generality.

In our previous papers, we proposed the general procedure of the new paradigm of solutions, the coupling scheme between overlapping elements and traditional finite elements, and a new overlapping element that is distortion insensitive and not expensive in the numerical integration. However, the overlapping element leads to a much larger bandwidth than the traditional finite elements.

In this paper, we focus on a significant improvement of the overlapping element of Ref. [12] and evaluate the computational efficiency of the new paradigm of solutions when using this element. First, in Section 2 we present the formulation of the improved overlapping element, which gives a much smaller bandwidth than the element in Ref. [12], and we discuss the required numerical integration and the element insensitivity to geometric distortions. Then in Section 3, we study the computational efficiency of the new overlapping element; namely in the required numerical integration and in the solution of the equations in comparison to using the traditional 4-node finite elements. We also study the convergence properties of the overlapping finite element. Thereafter, in Section 4, we illustrate the complete analysis approach of the new paradigm in the analysis

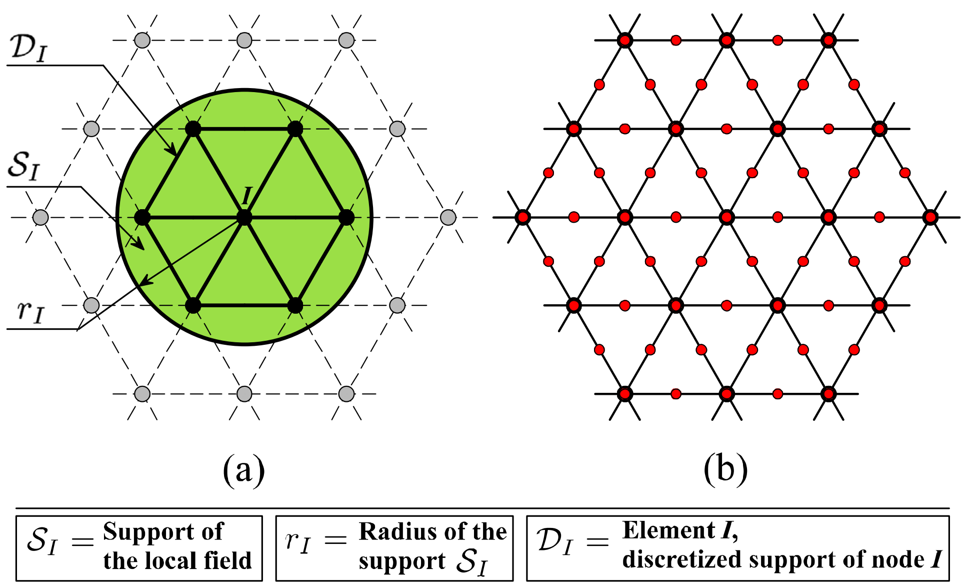

Fig. 1. Schematic of the new overlapping element with triangular overlap regions; (a) the 7-node overlapping element $\mathcal{D}_I$; (b) the mesh of the overlapping elements with red virtual nodes included. (To see the colors in this figure and all subsequent figures, refer to the web version of the paper.)

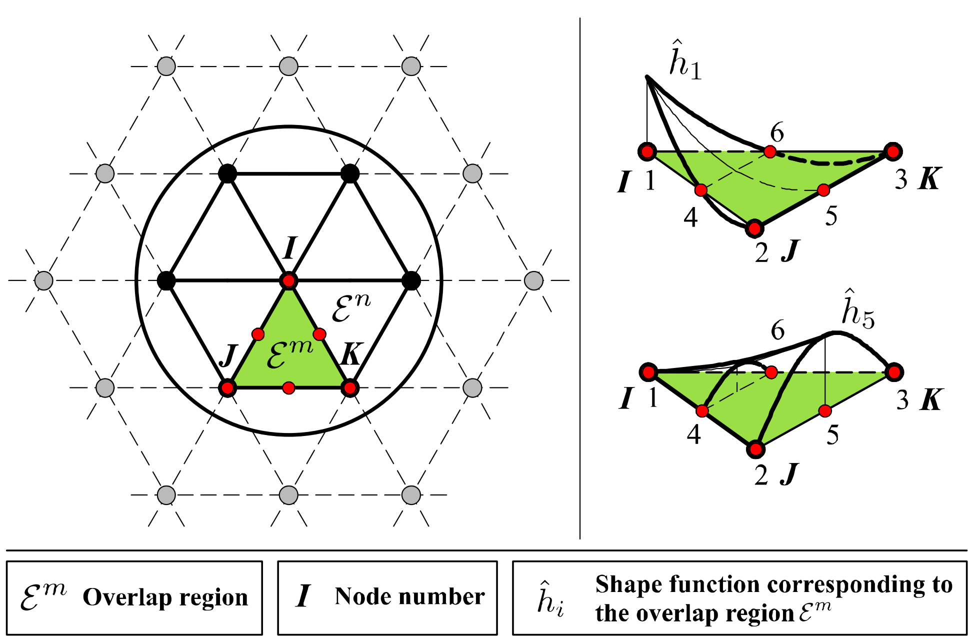

Fig. 2. Schematic of shape functions $\hat{h}_1$, $\hat{h}_5$ corresponding to the overlap region $\mathcal{E}^m$; there are 6 virtual nodes for $\mathcal{E}^m$, numbered from 1 to 6.