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Transient implicit wave propagation dynamics with overlapping finite elements

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ABSTRACT

We present novel overlapping finite elements used with the Bathe time integration method to solve transient wave propagation problems. The solution scheme shows two important properties that have been difficult to achieve in the numerical solution of general wave propagations: monotonic convergence of calculated solutions with decreasing time step size and a solution accuracy almost independent of the direction of wave propagation through the mesh. The proposed scheme can be efficiently used with irregular meshes. These properties make the scheme (the combined spatial and temporal discretizations) promising to solve general wave propagation problems in complex geometries involving multiple waves. A dispersion analysis is given and various example problems are solved to illustrate the performance of the solution scheme.

1. Introduction

The finite element method with direct time integration is widely employed to solve transient wave propagation problems. Using the traditional finite element solution approach, however, accurate solutions are difficult to obtain due to the dispersion and dissipation errors caused by the spatial and temporal discretizations, see for example Refs. [1–7]. Accurate solutions can only be obtained of rather simple problems, like one-dimensional problems with a single wave traveling through the domain. In this case, a uniform mesh and optimal time step size can be used. However, for geometrically complex problems, irregular meshes need in general be used and it is difficult to improve the solution accuracy by refining the mesh and decreasing the time step size, whichever spatial and time discretizations are used. For such irregular meshes, the solution accuracy depends on the propagation direction considered even when the wave is traveling through an isotropic medium. The traditional finite element method with direct time integration is, therefore, not very effective for the solution of general two- and three-dimensional wave propagation problems with waves propagating in different directions and at different wave speeds.

Considerable research efforts have focused on reducing the dispersion error of finite element solutions, see for example Refs. [8–12]. Also, the spectral element method can be used [13,14]. However, the above difficulties have not been overcome when considering complex geometries, anisotropic media, general boundary conditions and multiple waves traveling through the continuum.

The method of finite spheres, a meshless method, enriched for wave propagation problems can be used with the Bathe time integration scheme to solve wave propagation problems but *uniform* spatial discretizations need be used [15,16]. An important observation in Refs. [15,16] is that in the uniform spatial discretizations, a decrease in the time step size leads to a more accurate solution, which is what an analyst intuitively expects, and numerical anisotropy is almost negligible. These are important observations because by using the largest wave speed to establish the time step size, accurate solutions for multiple types of waves can be obtained *and* regardless of the propagation directions. The details of the mathematical analysis of the solution procedure and illustrative example solutions are given in Refs. [15,16].

However, the major difficulty in using the method of finite spheres, like other meshless methods, is the very expensive numerical integration for the construction of the mass and stiffness matrices [17–19]. The integration cost is clearly prohibitive for irregular discretizations using spheres, see Ref. [19]. For uniform discretizations, the numerical integration can be performed only once for a typical sphere and the result can then be reused [18], but this approach can of course not be employed when non-uniform spatial discretizations need be used. The high computational cost of the method impedes its wide practical use in industry.

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Recently, we proposed a new paradigm of analysis using mostly traditional finite elements with some overlapping finite elements, like finite spheres [19–22]. We also developed novel overlapping finite elements and demonstrated that for static analysis, using these overlapping finite elements, the solution accuracy is almost insensitive to geometric distortions and the numerical integration is not very expensive compared to the use of traditional finite elements [22]. While we considered only static solutions, the use of overlapping finite elements has clearly also good potential for dynamic analyses.

In this paper, for the solution of transient wave propagation problems, we enrich the overlapping finite elements of Ref. [22] using trigonometric functions and use the Bathe time integration method because of its favorable dissipation properties [23,24]. The same approach has already been applied for use of a traditional finite element [25] and the method of finite spheres [15,16]. However, as already mentioned above, the use of the method of finite spheres is not efficient in general practical analyses because of the very expensive numerical integrations. For the traditional finite element enriched with trigonometric functions, the solution effort is more acceptable, although high, but the solution accuracy is not as desired because the predicted response sensitively depends on the directions of waves traveling through the mesh and fine meshes or high-order harmonic functions are required.

Our objective in this paper is to analyze the overlapping finite element enriched with trigonometric functions together with the Bathe time integration method and illustrate that the combined spatial and time discretization scheme can be used to solve wave propagations in complex geometries using regular or irregular meshes. Hence, as we also demonstrate, the element can be used with the new paradigm of finite element solutions for CAD.

In the next section, we formulate the overlapping finite element for transient wave propagation problems. Then, in Section 3, we study the dispersion properties of the proposed scheme. Thereafter, in Section 4, we provide the calculated solutions of various wave propagation problems to illustrate the capability of the solution scheme. We focus on showing that even when using irregular meshes good results are obtained. Finally, we give the conclusions of our research in Section 5.

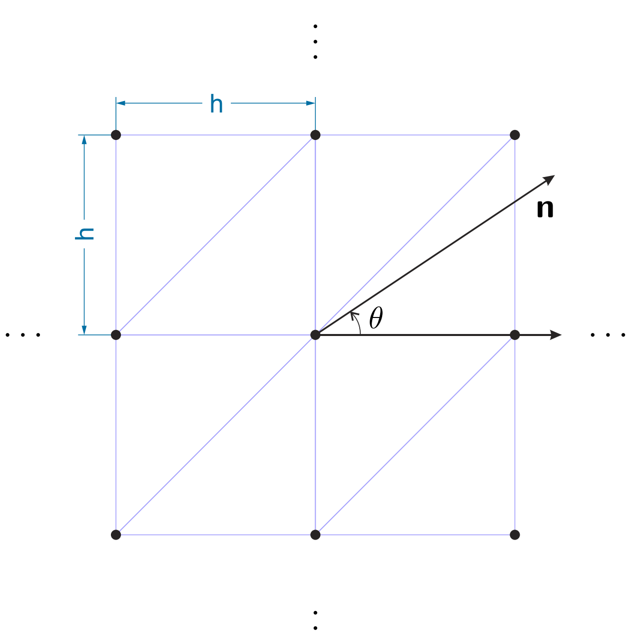

Fig. 1. Uniform mesh and propagation angle of a sinusoidal plane wave.

2. Spatial approximation scheme

In the new paradigm of finite element analysis, the global analysis domain is discretized by traditional finite elements (that do not overlap) and finite elements that overlap [20,21]. For every overlap region, the solution variable u is approximated as [19,22]

$$u \approx u_h = \sum_{I=1}^{q} h_I \psi_I = \sum_{I=1}^{q} h_I \sum_{J \in \mathcal{N}_I} \hat{\varphi}_J^I \sum_{n \in \mathfrak{I}_J} p_n a_{Jn} \tag{1}$$

where q is the number of nodes in the overlap region, h_I is the shape function used in the traditional finite element [26], ψ_I is the local field of the overlapping element I, $\mathcal{N}_I$ is the set of nodes located in the overlapping element I, $\hat{\varphi}_J^I$ is a partition of unity function, $\mathfrak{I}_J$ is an index set and p_n is a set of local basis functions (e.g., a polynomial for elliptic problems) which span the local approximation space V_J^h with the corresponding coefficient of node J. It is important to note that the function $\hat{\varphi}_J^I$ is a polynomial and hence the computational cost for establishing the stiffness and mass matrices is not high.

For the solution of two-dimensional wave propagation (hyperbolic) problems, the bi-linear polynomials and trigonometric functions (used like polynomials) are employed for the local approximation space, i.e., at node J we use

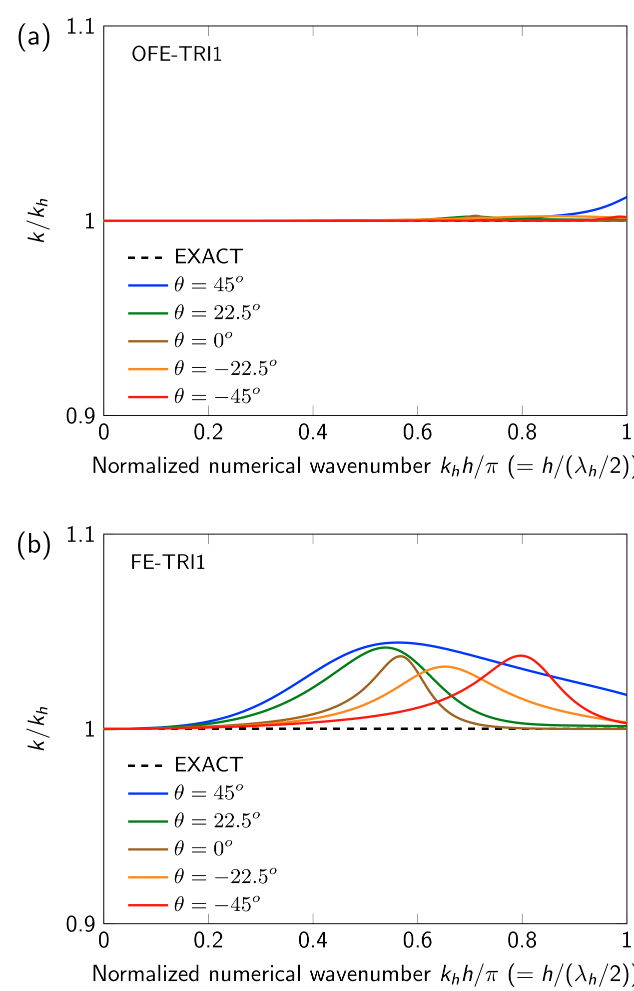

Fig. 2. Dispersion properties of (a) OFE-TRI1 and (b) FE-TRI1 discretizations for various propagation angles.