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Topology optimization of deformable bodies with dissimilar interfaces

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ABSTRACT

The topology optimization for practical engineering problems is computationally expensive owing to the complexity of the entire system. Therefore, most of the topology optimization is currently being conducted on simplified decomposed subsystems, which are then assembled in order to reduce the computational cost. Under these circumstances, there is a possibility that an inappropriate design might be obtained from the overall system. To overcome this limitation, an accurate and efficient algorithm for performing the structural topology optimization of deformable bodies containing dissimilar interfaces is introduced. Based on the mortar method, the condensed mortar method is proposed to connect dissimilar interface boundaries and to handle them in a manner similar to that used in conventional structure analysis. In this way, the treatment of such a problem becomes very concise, and the computational cost can be significantly reduced. Furthermore, the topology optimization is implemented using a modified SIMP method to derive the most suitable optimum layout. For alleviating the numerical deficiency at the interfaces, appropriate filtering schemes are adopted. Finally, several numerical examples are presented to verify the validity of the proposed method.

1. Introduction

In a variety of engineering problems, several types of research focus on deriving optimum layouts of structures at a conceptual level of the design process using topology optimization. However, in several practical applications, the entire system is composed of several subsystems that are modeled separately to construct an efficient system such as local refinement for the purpose of each problem. Moreover, when performing a variety of nonlinear analysis including multi-material, multi-physics, as well as contact problems, the finite element models should be discretized independently. In these circumstances, the analysis and optimization might be performed individually, and the results are then collated. Because of this, imposing precise boundary conditions appropriately along the interface is an important factor that significantly affects the performance of the analysis and optimization.

Recently, several types of research have been conducted to treat these complicated systems. In a fluid-structure interaction problem that is coupled with various governing equations, the topology optimization for a solid structure is conducted using monolithic formulation [1]. As for the contact problems, topology optimization for frictionless contact is performed while considering the geometric nonlinear effect [2,3]. However, the previous researches have been performed on single structures. In order to proceed with topology optimization for structures consisting of more than one structure, a study was carried out on a system with two elastic subsystems [4]. In this study, a node-to-node contact algorithm is formulated to connect two subsystems after coinciding the interface nodes. However, this method is limited to the problems with matching meshes only. Therefore, in order to address a greater number of general engineering problems, this paper focuses on a new approach that derives an optimum result for systems with dissimilar interfaces.

For treating a system with dissimilar interfaces, the mortar method is the most popular approach that can be used to impose interface compatibility conditions. The method enforces the interface conditions using Lagrange multipliers that act as interface forces or fluxes in a variational sense. Depending on how the interface compatibility conditions are defined, the mortar method is categorized into two types: classical Lagrange multipliers (CLM) and localized Lagrange multipliers (LLM) versions. In the CLM method, the interface conditions are imposed in a weak form using Lagrange multipliers introduced as independent third variables [5–7]. The weak imposition can be used to derive reasonable results even when the interfaces are deformed [8]. From a computational point of view, the CLM method yields two types of linear systems of equations: a linear system with Lagrange multipliers [5–8] and a

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linear system without Lagrange multipliers [9,10]. For numerical efficiency and accuracy, the latter is preferred owing to the characteristics of the total system equations. For confirming the numerical features of the systems, several efficient linear solvers are studied with verification examples [11,12]. As with the CLM method, the LLM method has been studied for various interface problems [13–16]. A distinctive characteristic of the LLM comparing with the CLM is the creation of a fictitious frame domain among the interface boundaries for imposing a unique set of constraints [14]. Owing to the key features of the LLM method, it has found application not only in interface and contact problems possessing same constitutive models but also in a host of coupled-field problems such as fluid-structure interaction problems [17–19].

The topology optimization derives an optimal layout by eliminating unnecessary parts of the structure [20]. The basic procedure involves the elimination of some areas of the design domain that have no contributions for optimizing the objective function and satisfying the constraints. In other words, the optimal layout of the topology optimization should have the essential parts of the desired performance [20,21]. Therefore, the creative and aesthetic solution can be obtained at a conceptual level of the design process. Moreover, it is convenient to formulate and implement [22]. For these reasons, topology optimization has been developed as a strong, powerful, and versatile tool for various design applications since the late 1980s [23]. The most widely used algorithm is the solid isotropic material with penalization (SIMP) method [20,22,24]. The relative density is defined at each element as a design variable for expressing material properties during analysis and optimization [22]. However, the SIMP method unfortunately has several numerical shortcomings such as checkerboard pattern, mesh-dependency, and intermediate density. In order to overcome these issues, several schemes including a perimeter constraint, gradient constraint, and filtering techniques have been proposed [25]. Among these, the filtering scheme is commonly applied as it results in appropriate optimum solutions as compared to other techniques. For adopting the filtering schemes more easily and efficiently, the modified SIMP method is developed by employing a refined interpolation scheme that is independent of the penalization factor [26].

The rest of this paper is organized as follows. Section 2 provides an overview of the conventional mortar methods including the classical and localized versions. Based on the conventional methods, the condensed mortar method is proposed by eliminating the Lagrange multipliers and overlapped interface displacements to modify the system equation as a conventional structural system. In Section 3, the formulations for the modified SIMP method are introduced. Then, a new topology optimization methodology is adopted to obtain optimal solutions for structures containing dissimilar interfaces. In order to verify the validity of this approach, a simple example is solved for comparison with the results obtained using a commercial software program, ABAQUS. In Section 4, several numerical examples comprising two substructures are used to verify the accuracy and effectiveness of the present method. Finally, the concluding remarks are presented in Section 5.

2. Condensed mortar method

This section presents a streamlined review for the conventional mortar methods including classical and localized versions for connecting boundaries along a dissimilar interface. Then, the conventional method is modified to obtain the proposed condensed mortar method.

A simple interface patch test problem that consists of two elastic bodies, as depicted in Fig. 1, is considered to demonstrate the characteristics of the mortar and condensed mortar method. Schematics of the mortar methods for CLM and LLM are shown in Fig. 1(b) and (c), respectively. As shown in Fig. 1(c), the LLM method is implemented by considering a fictitious frame domain to impose the interface conditions properly.

2.1. Review of the conventional mortar method

Depending on how the interface compatibility conditions and Lagrange multipliers are defined, the mortar method is divided into the CLM and LLM methods. The CLM method is the most popular approach for treating interface conditions by introducing the Lagrange multipliers as the interface pressure shown in Fig. 1(b). In the LLM method, a fictitious frame is introduced and the interface condition is imposed indirectly using the frame shown in Fig. 1(c) [13]. Using the Lagrange multipliers, a subsystem is connected to an adjacent subsystem while satisfying the interface conditions.

When the finite element approaches are adopted, the hybrid form of a total energy functional has to be defined for deriving the formulations of the mortar method [14].

$$\Pi_{\mathrm{TPE}} = \sum_{\Omega=1}^{N} \Pi_{\mathrm{PE}}^{\Omega} - \Pi_c \tag{1}$$

where Π_{TPE} is the total energy, Π_c is the interface potential energy, N is the number of subdomains (in Fig. 1, $N = 2$), and $\Pi_{\mathrm{PE}}^{\Omega}$ is the potential energy for each subdomain Ω in which it is given as

$$\Pi_{\mathrm{PE}}^{\Omega} = \int_{\Omega} [\upsilon(u) - u_i \bar{f}_i] d\Omega - \int_{\partial\Omega} u_i \bar{t}_i d\Gamma. \tag{2}$$

In Eq. (2), the potential energy comprises the internal energy expressed in terms of the strain energy density function $\upsilon(u)$, which is given by

$$\upsilon(u) = \frac{1}{2} D_{ijkl} \varepsilon_{ij} \varepsilon_{kl}, \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

and the external energy is obtained from the body force $\bar{f}$ and surface traction $\bar{t}$, where u is a displacement field, D_{ijkl} is the fourth-order elastic Young's modulus, the conventional summation rule is in effect, and the comma represents partial derivatives.

2.1.1. Method of Classical Lagrange Multipliers (CLM)

When the CLM method is applied, the interface potential energy Π_c is defined in order to impose the interface compatibility condition using Lagrange multipliers as shown in Eq. (4).

$$\Pi_c = \int_{\Gamma_I} \boldsymbol{\lambda}^T (\mathbf{u}^{(1)} - \mathbf{u}^{(2)}) d\Gamma \tag{4}$$

where Γ_I represents the interface boundary, $\boldsymbol{\lambda}$ the Lagrange multipliers, and $\mathbf{u}^{(1)}$ and $\mathbf{u}^{(2)}$ the displacement field for each subdomain, respectively. If the meshes of each subdomain along the interface do not completely coincide with the Γ_I, gaps are created. In that case, the interface compatibility condition is imposed after projecting the $\mathbf{u}^{(1)}$ and $\mathbf{u}^{(2)}$ on the Γ_I.

On substituting Eq. (4) into the total energy functional (1) and considering the first variation to the functional (1) for minimizing total potential energy, a matrix form of the total system for the CLM can be obtained as follows:

$$\begin{Bmatrix} \delta\mathbf{u}^{(1)} \\ \delta\mathbf{u}^{(1)} \\ \delta\boldsymbol{\lambda} \end{Bmatrix}^T \left(\begin{bmatrix} \mathbf{K}^{(1)} & 0 & \mathbf{B}_1\mathbf{Q}_1 \\ 0 & \mathbf{K}^{(2)} & -\mathbf{B}_2\mathbf{Q}_2 \\ \mathbf{Q}_1^T\mathbf{B}_1^T & -\mathbf{Q}_2^T\mathbf{B}_2^T & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{u}^{(1)} \\ \mathbf{u}^{(2)} \\ \boldsymbol{\lambda} \end{Bmatrix} - \begin{Bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \\ 0 \end{Bmatrix} \right) = 0, \tag{5}$$

$$\mathbf{Q}_1 = \int_{\Gamma_I} \mathbf{N}_{u_{\Gamma 1}}^T \mathbf{N}_{\lambda} d\Gamma, \quad \mathbf{Q}_2 = \int_{\Gamma_I} \mathbf{N}_{u_{\Gamma 2}}^T \mathbf{N}_{\lambda} d\Gamma$$