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Stress-constrained topology optimization based on maximum stress measures

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ABSTRACT

This paper proposes two effective constraint schemes to address the stress-constrained topology optimization of continuum structures. By considering the maximum stress measure in the global and local forms, respectively, the STM (stability transformation method)-based stress correction scheme and the violated set enhanced stress measure are developed to tackle the challenging issues from numerous local stress constraints and highly nonlinear stress behavior. Particularly, a stress aggregation function is involved in the design sensitivity analysis. Moreover, the nodal variable based SIMP method and adjoint sensitivity analysis are employed to solve the optimum topological design problems with two different optimization formulations. Finally, several representative examples demonstrate the validity of the present approach. It is also indicated that the numerical performance of the stress aggregation function is closely related to the problem formulation of topology optimization. The STM-based stress correction scheme is appropriate to the material volume minimization design, while the violated set enhanced stress measure is suitable for the mean compliance minimization design. Meanwhile, the proposed optimization approach can handle the stress-constrained topology optimization with easy implementation, low computational cost and stable convergence.

1. Introduction

Topology optimization is a conceptual design tool for optimizing the material layout of a structure within the given design space, which possesses greater design freedom than size and shape optimization. Since the pioneering work of [1], there have been rapid development and extensive applications for topology optimization of continuum structures over the past two decades. Nowadays, various methods are available to address the topology optimization design problem in many engineering fields. Notably, the density-based SIMP (solid isotropic material with penalization) method has been popularly applied owing to the conceptual simplicity, easy implementation and computational efficiency. For an overview of topology optimization approaches and their applications, the readers are referred to the monograph [2] and the review articles [3–6].

Earlier researches of topology optimization were mostly devoted to the compliance minimization design problem, which is intended to enhance the overall stiffness of structure. However, the strength index is paramount from the viewpoint of engineering application. In recent years, much attention has been paid to the topology optimization problems involving stress-related objective or constraints because of their importance and complexity [7–15]. As pointed out in [6], compared with the minimizing compliance design, there are three challenging issues imposed by this kind of problem, such as the stress singularity phenomenon, the local nature of stress constraints and the highly non-linear stress behavior.

First, the stress singularity arises in the density-based optimization approaches due to the discontinuity of local stress constraints, when topology design variables tend to the critical values. It may prevent optimization algorithms from finding the true optimum design [2]. To avoid falling into the singular optimum, some relaxation methods were proposed such as the epsilon-relaxation method [16,17], the *qp* relaxation method [18], and the stress penalization method [8]. Second, the local nature of stress constraints could result in huge computational burden for sensitivity analysis. Accordingly, the stress aggregation functions were suggested to reduce the computational efforts in [19–21]. Therein, the local stress constraints are often replaced with some stress aggregation formulation, e.g., the P-norm or the Kreisselmeier-Steinhauser (K-S) stress measure. Although such global aggregation methods are computationally efficient, the optimized results

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are not acceptable sometimes. Thus, some regional or clustered aggregation techniques are proposed to make a trade-off between the global and local stress constraint methods [8,22–24]. Third, the stress constraints are highly nonlinear and the stress levels are drastically affected by density change and local geometric property, especially in the structural boundary with high curvature. There are large stress gradients in these regions such as reentrant corners with stress concentration. Hence, the stress concentration regions should be captured adequately and subsequently dealt with by appropriate optimization formulation and solution algorithm, which is important for overcoming convergence difficulty of topology optimization [7,8,12].

With much research efforts into the stress-based topology optimization, there is more and more in-depth understanding for such problem, which makes remarkable progress recently [6]. Some works focused on the treatment of local stress constraints by introducing efficient schemes, so as to avoid the possible stress concentration and relax the high stress levels. Generally, an effective aggregation scheme is desirable, if it can handle the stress-constrained topology optimization with easy implementation and low computational cost. Despite the obvious increase in solution efficiency, the traditional P-norm or K-S aggregation method is difficult to enforce an equivalent constraint on the maximum stress of structure. As a consequence, it usually leads to poor local control over the stress distribution and a serious violation of some stress constraints. Moreover, the high nonlinearity of these aggregation stress functions often causes unstable convergence during the optimization process.

In view of above drawbacks of the traditional aggregation methods, some improved strategies and new stress measures were proposed to tackle the challenging stress constraint issues. Some of the representative schemes include the normalized P-norm measure approach [8], the effective penalty functional approach [9], the shape equilibrium constraint strategy [12], the global stress measure method [13] and several recently developed numerical techniques [15,23–27]. With use of these constraint schemes, the stress-constrained volume minimization design or compliance minimization design was conducted to show their validity. Although the existing approaches work for alleviating the severe stress concentration and obtaining satisfactory topological design, it is still an open problem to establish new stress constraint formulations, which can effectively handle the numerous local stress constraints and avoid the possible convergence difficulty. In addition, the numerical performance of a stress constraint scheme to the topological optimization problem formulation needs further investigation.

This paper aims to develop two effective constraint schemes for performing the maximum stress constraint in continuum structural topology optimization. To this end, two types of constraint formulation are suggested based on the maximum stress measures with the global and local forms, respectively. One is based on the correction of global stress measures (e.g., P-norm and K-S stress measure), in conjunction with the stability transformation method (STM) originally for chaos control of nonlinear dynamical system and oscillation control of iterative computation [28–30], namely the STM-based stress correction scheme. Another is based on the maximum local stress with an enhancement of the violated stress set, namely the violated set enhanced stress measure. Moreover, the topology optimization problem is solved by employing the nodal variable based SIMP method and adjoint sensitivity analysis. The design variables are updated by the method of moving asymptotes (MMA) [31]. Finally, several representative examples of 2D structures in plane stress demonstrate the stable convergence and desired optimum design of the proposed stress constraint schemes.

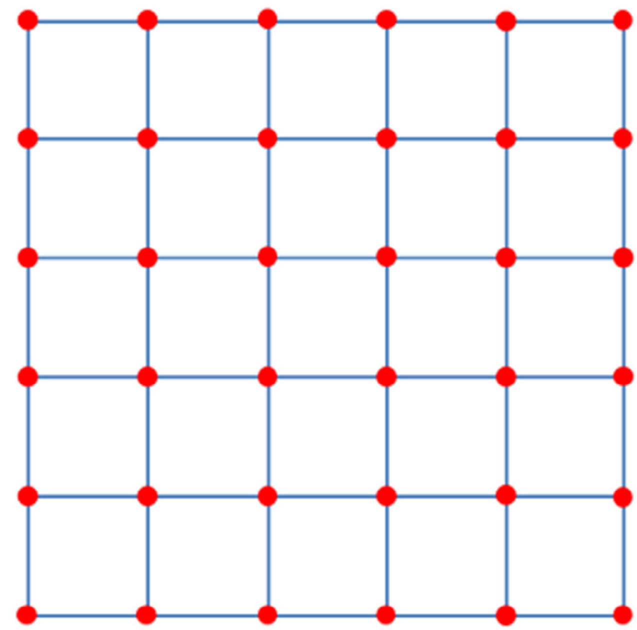

Fig. 1. Bilinear four-node square elements.

The remainder of this paper is organized as follows. In Section 2, stress-constrained topology optimization problem is formulated for the lightest design and the stiffest design of continuum structure with numerous local stress constraints, respectively. Two stress constraint schemes for limiting the maximum stress are proposed in Section 3, i.e., the STM-based stress correction scheme and the violated set enhanced stress measure. In Section 4, the adjoint sensitivity analysis formulation is presented for stress-constrained topology optimization. In Section 5, the effectiveness of the proposed stress constraint schemes is demonstrated by several benchmark tests. Main conclusions are finally drawn in Section 6.

2. Formulations of stress-constrained topology optimization

This section presents the topology optimization formulations with local von Mises stress constraints based on the nodal density variable based SIMP method. Two optimization formulations are considered, namely the stress-constrained volume minimization design and the mean compliance minimization design with both volume and stress constraints.

2.1. Nodal variable based SIMP model

The density-based SIMP method is popularly used by topology optimization researchers, which has been applied to a wide range of industrial design problems [5,6]. For simplicity, the bilinear (Q4) finite elements combined with the nodal variable strategy are employed in the present work. And the nodal density interpolation scheme with element-wise constant densities is utilized to achieve checkerboard-free solutions [32–35]. As stated in the review paper [5], the intrinsic mapping between nodal variables and elemental densities is a filter in itself and hence can prevent the formation of checkerboard patterns.

As shown in Fig. 1, the four-node square elements are employed for both analysis discretization and design parameterization, in which the density variables are located at the red nodes. Therein, a constant density distribution within each element is adopted,