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Model reduction schemes for the wave and finite element method using the free modes of a unit cell

Y. Fan [a,b,d], C.W. Zhou [b], J.P. Laine [b], M. Ichchou [b], L. Li [a,c,d,*]

[a] School of Energy and Power Engineering, Beihang University, Beijing 100191, China
[b] Laboratoire de Tribologie et Dynamique des Systemes, Ecole Centrale de Lyon, Ecully 69130, France
[c] Collaborative Innovation Center for Advanced Aero-Engine, Beijing 100191, China
[d] Beijing Key Laboratory of Aero-engine Structure and Strength, Beijing 100191, China

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ABSTRACT

The wave and finite element method (WFEM) is an efficient numerical tool for analysing wave propagation characteristics and forced response at intermediate and high frequencies. In this work, we introduce free-interface component modal synthesis (CMS) methods into WFEM to accelerate the calculation while maintaining the accuracy. Several free-interface CMS methods with different approximations of the residual effects are implemented and compared. A new eigenvalue scheme based on the dynamic compliance matrix is proposed. A periodic open thin-wall structure is considered as an application for which both free-wave characteristics and forced responses are computed. Aspects such as accuracy, efficiency, and convergence of the proposed method are discussed and compared with those of the Craig-Bampton fixed-interface CMS method. The methods and main findings are further verified by using another more complex periodic structure. Among the implemented models, the minimum model size is achieved by the exact CMS method. The exact CMS method only requires the modes below the maximum analysing frequency, thereby reducing the model size of the open thin-wall structure from 4416 to 16. The most numerically efficient model for WFEM is MacNeal's CMS method, where the CPU time of free-wave analysis can be reduced by 97% for the open thin-wall structure.

1. Introduction

Periodic structures are extensively encountered in many fields of engineering. Some such structures are as those involved in chassis frames and aircraft fuselages (one-dimensional), or honeycomb sandwiches, stiffened panels, and beam lattices (two-dimensional). These structures are often optimized to provide high structural integrity with low weight. The vibro-acoustic characteristics are therefore of utmost significance for these structures to avoid problems related to fatigue and noise, especially at intermediate and high frequencies where the modal density is high. Several methods [1–3] were proposed to analyse the dynamics of structures. Examples of these methods are the statistical energy analysis, finite element method, and wave-based methods. Among them, a wave-based method termed the wave and finite element method (WFEM) has gained increasing research interest [4,5]. The main feature of the WFEM is the introduction of the FE model of only one unit cell into the general principle of periodic structures [6], instead of considering the entire FE model of a whole periodic structure. This allows the analysis of complex engineering periodic structures with relative ease and low computational cost. The method has been applied to analyse pipes [7], curved members [8], thin-wall structures [9], piezoelectric composites [10], and built-up structures [11].

The WFEM is centred on the wave basis formed by the eigenvalues and eigenvectors of the transfer matrix of one unit cell. Because a FE unit cell model is used, a refined mesh is necessary to predict the wave characteristics well, as reported by Droz et al. [12]. However, the use of large FE models worsens the existing numerical problems [13]. All the existing eigenvalue schemes are based on the condensed dynamic stiffness matrix (DSM), which is obtained by eliminating all the inner DOFs of a unit cell. A large sparse matrix corresponding to the inner DOFs is inverted, and it may induce numerical errors into the condensed DSM; these errors cannot be reduced by using an appropriate eigenvalue scheme. Moreover, the condensed DSM is frequency-dependent, implying that the inverse of a big sparse matrix is required at each frequency. This could increase the computational cost drastically when a large FE model is used. On the other hand, the size of the eigenvalue problem is directly related to the number of DOFs at the boundaries. The use of a large FE model with more boundary DOFs will

* Corresponding author.
E-mail address: feililin@buaa.edu.cn (L. Li).

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also increase the computational cost for solving the eigenvalue problem.

To accelerate the calculation of the wave basis and to mitigate the numerical error, reduced models were proposed. Literature reports two main strategies to reduce a unit cell model for WFEM. In this paper, we label them as wave-based [14,12] and mode-based models [11,15,16]. The former strategy is preferable for uniform structures with complex cross-sectional profiles, while the latter is preferable for periodic structures with numerous inner DOFs.

The wave-based strategy employs a subset of wave shapes at some pre-selected frequencies to express the cross-sectional deformation at the present frequency. By adopting this strategy, the DOFs at the boundaries and the magnitude of the eigenvalue problem were reduced. To start this method, however the wave solutions at the pre-selected frequencies should be computed using the full FE unit cell model, which increases the implementation difficulty.

Alternatively for the mode-based strategy, a unit cell model is reconstructed by component modal synthesis (CMS) before the WFEM procedure, so that the size of the eigenvalue problem is reduced. CMS approaches are well established [17,18] and it has already been used in combination with the periodic structure theory [19] to enhance the statistic energy analysis method. For WFEM, Mencik [15] proposed the use of the Craig-Bampton CMS method [20] for unit cell modelling. Zhou et al. [16,21] applied the method for two-dimensional structures and experimentally validated the results. Fan et al. [11] extended this method to structures with local dampers or piezoelectric shunts. The accuracy of this strategy is ensured by the principle of modal superposition. Further, the accuracy can be improved by retaining more modes. It is intuitive to combine the Craig-Bampton method with WFEM because the boundary DOFs at which the periodic boundary conditions apply are retained in the physical domain. However, it is unclear whether the free-interface CMS method, another widely used CMS method, can also be applied to WFEM. If yes, such a combination can be used as an alternative way to build reduced models for periodic structures. In addition, it is easier to include experimental data into the reduced model for the free-interface CMS approach.

In this paper, we explore the use of the free-interface CMS methods with the WFEM. The basic idea of free-interface CMS methods is to use low-order free modes and residual effects to approximate the dynamic compliance matrix. Thus, a new eigenvalue scheme based on the force vector is proposed (Section 3.3), and the results can be easily recovered to the eigen-solutions of the transfer matrix. Free-interface methods proposed by Hou [22], MacNeal [23], Rubin [24], and Qiu et al. [25] were implemented (Sections 3.1 and 3.2). They have different orders of accuracy for the residual effects from zero order to infinite. As references, the full WFEM (Section 2.1) and WFEM with the Craig-Bampton method were also implemented. A periodic thin-wall structure with complex wave characteristics was considered as the application (Section 4). The free-wave results obtained by the implemented methods were compared to illustrate the efficiency, convergence, and accuracy of the methods (Section 4.1). For the forced response analysis, the accuracy on the strong evanescent waves and their influences were discussed (Section 4.2). Finally, the methods and their efficiency were further verified by considering a more complex thin-wall structure.

2. Framework of the wave and finite element method (WFEM)

For the sake of clarity, we briefly review the WFEM for the free wave and forced response analysis. Obtaining the finite element description of a single unit cell is the starting point of WFEM. A unit cell is the smallest repetitive substructure of a periodic structure, as shown in Fig. 1. By imposing the periodic boundary conditions derived from the Bloch theorem, the homogeneous problem of the periodic structure results in an eigenvalue problem, whose scheme can be formulated in many different ways. The solutions yield wavenumbers and associated wave shapes at each frequency, revealing how free waves can travel in a structure. Additionally, the obtained left and right eigenvectors define the wave basis [26]. The wave basis has many useful properties that enable diagonalisation of the transfer matrix by a reduced set of left and right eigenvectors [27,28]. The forced response of the structure subject to external forces can then be obtained by wave decomposition and superposition [13,29].

2.1. WFEM with full FE model of a unit cell

In the context of free-wave analysis, external loads are not considered. After isolating a unit cell from the periodic structure, the discrete governing equations can be obtained by the existing FE tools:

$$\mathbf{M}\begin{pmatrix}\ddot{\mathbf{q}}_{\mathrm{L}}\\ \ddot{\mathbf{q}}_{\mathrm{R}}\\ \ddot{\mathbf{q}}_{\mathrm{I}}\end{pmatrix}+\mathbf{C}\begin{pmatrix}\dot{\mathbf{q}}_{\mathrm{L}}\\ \dot{\mathbf{q}}_{\mathrm{R}}\\ \dot{\mathbf{q}}_{\mathrm{I}}\end{pmatrix}+\mathbf{K}\begin{pmatrix}\mathbf{q}_{\mathrm{L}}\\ \mathbf{q}_{\mathrm{R}}\\ \mathbf{q}_{\mathrm{I}}\end{pmatrix}=\begin{pmatrix}\mathbf{f}_{\mathrm{L}}\\ \mathbf{f}_{\mathrm{R}}\\ \mathbf{0}\end{pmatrix} \tag{1}$$

where $\mathbf{q}$ is the vector of nodal displacement; $\mathbf{f}$ is the force vector; a superimposed dot denotes the derivative with respect to time; and $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ refer to the mass, damping, and stiffness matrices respectively. Subscripts L, R, and I, respectively, denote the left-side, right-side, and internal DOFs as illustrated in Fig. 1. With regard to harmonic motions, the dynamic equations of a unit cell at frequency ω are given by

$$\tilde{\mathbf{D}}\mathbf{q}=\left(-\omega^{2}\mathbf{M}+j\omega\mathbf{C}+\mathbf{K}\right)\mathbf{q}=\mathbf{f} \tag{2}$$

where $\tilde{\mathbf{D}}$ is the dynamic stiffness matrix.

According to the Bloch theorem, when a free wave travels in the periodic structure, the following conditions should be satisfied:

$$\mathbf{q}_{\mathrm{R}}=\lambda\mathbf{q}_{\mathrm{L}} \tag{3}$$

$$\mathbf{f}_{\mathrm{R}}=-\lambda\mathbf{f}_{\mathrm{L}} \tag{4}$$

where $\lambda=e^{-jk\Delta}$ describes the amplitude and phase changes when the wave travels from the left side to the right side of a unit cell. k is the wavenumber, and Δ is the length of the unit cell. The minus sign in Eq. (4) is induced by the equilibrium of the internal forces.

The objective of the free wave analysis is to find the nodal displacement vector $\mathbf{q}=\left(\mathbf{q}_{\mathrm{L}}^{\mathrm{T}}\quad\mathbf{q}_{\mathrm{R}}^{\mathrm{T}}\quad\mathbf{q}_{\mathrm{I}}^{\mathrm{T}}\right)^{\mathrm{T}}$ associated with a wavenumber k at frequency ω to satisfy Eqs. (1), (3) and (4). After eliminating all the internal DOFs $\mathbf{q}_{\mathrm{I}}$ from Eq. (1) at frequency ω, the condensed dynamic stiffness matrix of the unit cell is written as follows:

$$\begin{bmatrix}\mathbf{D}_{\mathrm{LL}} & \mathbf{D}_{\mathrm{LR}}\\ \mathbf{D}_{\mathrm{RL}} & \mathbf{D}_{\mathrm{RR}}\end{bmatrix}\begin{pmatrix}\mathbf{q}_{\mathrm{L}}\\ \mathbf{q}_{\mathrm{R}}\end{pmatrix}=\begin{pmatrix}\mathbf{f}_{\mathrm{L}}\\ \mathbf{f}_{\mathrm{R}}\end{pmatrix} \tag{5}$$

where

$$\begin{bmatrix}\mathbf{D}_{\mathrm{LL}} & \mathbf{D}_{\mathrm{LR}}\\ \mathbf{D}_{\mathrm{RL}} & \mathbf{D}_{\mathrm{RR}}\end{bmatrix}=\begin{bmatrix}\tilde{\mathbf{D}}_{\mathrm{LL}} & \tilde{\mathbf{D}}_{\mathrm{LR}}\\ \tilde{\mathbf{D}}_{\mathrm{RL}} & \tilde{\mathbf{D}}_{\mathrm{RR}}\end{bmatrix}-\begin{bmatrix}\tilde{\mathbf{D}}_{\mathrm{LI}}\\ \tilde{\mathbf{D}}_{\mathrm{RI}}\end{bmatrix}\tilde{\mathbf{D}}_{\mathrm{II}}^{-1}\left[\tilde{\mathbf{D}}_{\mathrm{IL}}\quad\tilde{\mathbf{D}}_{\mathrm{IR}}\right] \tag{6}$$

Substituting condition (3) into Eq. (5) to eliminate $\mathbf{f}_{\mathrm{L}}$ and $\mathbf{f}_{\mathrm{R}}$, and considering Eq. (4), it comes to the eigenvalue problem

$$\left(\begin{bmatrix}\mathbf{0} & \sigma\mathbf{I}\\ -\mathbf{D}_{\mathrm{RL}} & -\mathbf{D}_{\mathrm{RR}}\end{bmatrix}-\lambda\begin{bmatrix}\sigma\mathbf{I} & \mathbf{0}\\ \mathbf{D}_{\mathrm{LL}} & \mathbf{D}_{\mathrm{LR}}\end{bmatrix}\right)\begin{pmatrix}\mathbf{q}_{\mathrm{L}}\\ \mathbf{q}_{\mathrm{R}}\end{pmatrix}=\mathbf{0} \tag{7}$$