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A subinterval decomposition analysis method for uncertain structures with large uncertainty parameters

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ABSTRACT

A simple and efficient subinterval decomposition analysis method is proposed to evaluate the lower and upper bounds of structural responses with large uncertain parameters. The proposed method decomposes the original structural system with multi-dimensional interval parameters into multiple one-dimensional subsystems. Every subsystem has only one interval parameter and the other interval parameters are substituted by their midpoint values. By dividing the interval parameter of each subsystem into several subintervals with small uncertainty, the lower and upper bounds of the system are approximately calculated by only a few subinterval combinational analyses instead of all possible combinations of subintervals. Finally, the accuracy and efficiency of the proposed method compared with the first-order Taylor method, Chebyshev interval method and traditional subinterval method are verified by several numerical examples and applications.

1. Introduction

Uncertainty factors are ubiquitous in practical engineering problems and they are generally categorized into two parts. The first typical one is our well-known aleatory uncertainty which is the inner characteristic of a specific system and is impossible to be deleted or controlled. Another one is commonly known as epistemic uncertainty or subjective uncertainty. Specifically, this uncertainty occurs because we lack of some relevant information or knowledge to construct a precise model to measure the uncertain parameter. This uncertainty or imprecision could be gradually reduced with gaining more information or experimental data. Meanwhile, various uncertainty quantification models have been developed and mainly include three types. The first one is the probability model expressed by random variable [1–4] which obeys a given probability distribution requiring sufficient samples to be precisely constructed. The second one is fuzzy model expressed by the fuzzy variable which obeys a kind of membership function [5,6] and the last one is convex model including interval model [7–10].

The interval model only requires a small number of samples to be constructed which is well suitable for practical engineering problems. The interval analysis method was firstly proposed by Moore [11] and subsequently many improved interval analysis methods and their applications have been developed [12–14]. The interval arithmetic for interval analysis is the simplest approach to calculate the bounds of response function. However, it always leads to overestimation due to dependency assumption [15,16]. For this shortcoming, Hansen [17] proposed the generalized interval arithmetic to obtain a sharper boundary response. Elishakoff et al. [18–20] and Muscolino and sofi [21–23] developed a parameterized interval analysis (PIA) and an improved interval analysis by extra unitary interval (IIA-EUI) to achieve the sharper bounds than those calculated by the traditional interval analysis method, respectively. Santoro et al. [24] combined the PIA and the IIA-EUI to solve the set of governing algebraic interval equations. Based on the extra unitary interval, Sofi and Romeo [25] developed an improved interval finite element method to address the static analysis of linear-elastic structure with interval parameters. Jiang et al. [26] proposed a new interval arithmetic method considering the correlation between any two interval parameters which can significantly alleviated the overestimation. Furthermore, the affine arithmetic [27,28] was introduced as improvements of the traditional interval analysis to overcome this issue.

At the same time, Chen et al. [29], Qiu and Wang [30] and Qiu et al. [31] employed the interval perturbation or Taylor series expansion technique to evaluate the dynamic response of the nonlinear system. Sevillano et al. [32] developed the modal interval

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method to estimate damage structural problems with interval parameters. Wang et al. [33] employed the first order Taylor expansion method to calculate the structural response with small measurement data. Gao et al. [34,35] presented the interval factor method to calculate the dynamic responses of different trusses. However, it is required that the uncertainties are synchronized based on interval factor and the obtained responses may be relatively conservative. Fujita and Takewaki [36] proposed an enhanced and efficient interval analysis within the framework of an updated second order Taylor series expansion. Meanwhile, Wu et al. [37,38] utilized the Chebyshev interval method to calculate the boundary responses of the nonlinear dynamic systems. This method promoted the development of the interval analysis for solving the nonlinear function at some degree, but its computational cost is very high when the dimension of uncertain parameter is relatively large.

Most of the aforementioned works are mainly limited to the small uncertainty problems and the upper and lower bounds of structures are evaluated based on the first order perturbation or Taylor series expansion techniques which require calculating the partial derivatives of system function. Generally, these methods are difficult to predict the responses of structures with large uncertainty parameters. To address this issue, the subinterval analysis method, which divides the large uncertainty parameter into several subintervals with small uncertainty level, is utilized as a powerful and efficient tool to evaluate bounds of response function. Xia et al. [39] extended the interval and subinterval perturbation methods to solve the frequency responses of a structural-acoustic system. Wang et al. [40] and Wang and Qiu [41] employed subinterval analysis with high-order terms of Neumann series to solve the heat conduction problems with large uncertainty parameters. Chen et al. [42] applied the subinterval analysis technique with interval homogenization-based method to estimate of the effective elastic tensor for microscopic material properties with interval uncertainty. However, every possible combination of subintervals requires conducting two interval analyses, so that the computational cost of existing subinterval analysis increases exponentially and is very huge when the number of interval parameters increases to a relatively large constant. This issue hinders the subinterval analysis method from widely applying in uncertain structures with high-dimensional interval parameters. In this paper, a subinterval decomposition analysis method is developed to evaluate the upper and lower bounds of uncertain structures with large uncertainty parameters. The proposed method could deal with uncertain structures with high-dimensional and large uncertain parameters. The partial derivatives of response function are not required to be evaluated in the proposed method, which, however, are necessary to be calculated in the Taylor series expansion and the traditional subinterval method.

The remainders of this paper are organized as follows. Section 2 introduces the statement of uncertain problem and relevant concepts. The details of proposed algorithm are introduced in Section 3. Three numerical examples and three engineering applications are analyzed to verify the effectiveness of the proposed method in Section 4 and Section 5, respectively. Finally, Section 6 gives the conclusions of this paper.

2. Statement of the problem

Owing to the existing interval parameter vector $\mathbf{X}^I = (X_1^I, X_2^I, \ldots, X_n^I) \in I(\mathbf{R}^n)$, the response of a specific system will be an interval and the interval response Y^I can be formulated as:

$$Y^I = f(\mathbf{X}^I) \tag{1}$$

where $f(\cdot)$ is the objective function of a structure or system. The lower and upper bounds of the uncertain parameter $X_i, i = 1, 2, \ldots, n$ are denoted as X_i^L and X_i^U, respectively; n is the number of uncertain parameter. The superscripts I, L and U denote the interval, lower and upper bounds of uncertain parameter, respectively.

The interval parameter vector X_i^I can be written as [16,43]:

$$X_i^I = X_i^C + [-1, 1]X_i^W \tag{2}$$

where the superscripts C and W are the midpoint and radius of uncertain parameter, respectively. The midpoint and radius of uncertain parameter X_i^I are calculated by:

$$X_i^C = \frac{X_i^U + X_i^L}{2}, X_i^W = \frac{X_i^U - X_i^L}{2} \tag{3}$$

Meanwhile, the uncertain vector $\mathbf{X}^I$ can be also rewritten as:

$$\mathbf{X}^I = \mathbf{X}^C + \delta\mathbf{X} \tag{4}$$

where $\delta\mathbf{X} \in [-1, 1]\mathbf{X}^W, \delta X_i \in [-1, 1]X_i^W, i = 1, 2, \ldots, n$. The uncertainty level γ of the interval parameter X_i is defined as:

$$\gamma = \frac{X_i^W}{|X_i^C|} \times 100\% \tag{5}$$

For the interval parameter X_i with large uncertainty level, it is divided into several subintervals with small uncertainty level γ. The number of subintervals for the uncertain parameter X_i is m_i and the length or width of each subinterval ΔX_i is calculated by:

$$\Delta X_i = \frac{X_i^U - X_i^L}{m_i} \tag{6}$$

Then the traditional subinterval analysis is conducted in every combination of subintervals to calculate the response of a structure [44]. However, the computational cost of traditional subinterval analysis is very high resulting in unaffordable computational cost for complex structures when the number of uncertain parameter is relatively large. Hence, a subinterval decomposition analysis is developed to calculate the response of structures with large uncertainty parameters.

3. A subinterval decomposition analysis method

Assumed that the uncertainty level of the interval parameters are relatively small, the objective function $f(\mathbf{X})$ can be approximated in the uncertainty domain by the second-order Taylor expansion at the midpoint of uncertain parameter vector $\mathbf{X}^C$:

$$f(\mathbf{X}) = f(\mathbf{X}^C) + \mathbf{G}^{\mathrm{T}}(\mathbf{X}^C)\delta\mathbf{X} + \frac{1}{2}\delta\mathbf{X}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\mathbf{X}^C)\delta\mathbf{X} \tag{7}$$

where $\mathbf{G}^{\mathrm{T}}$ and $\mathbf{H}^{\mathrm{T}}$ are the gradient vector and the Hessian matrix, respectively. The matrix $\mathbf{G}^{\mathrm{T}}$ and $\mathbf{H}^{\mathrm{T}}$ are expressed as:

$$\mathbf{G}^{\mathrm{T}} = \left[\frac{\partial f}{\partial X_1}, \frac{\partial f}{\partial X_2}, \ldots, \frac{\partial f}{\partial X_n}\right] \tag{8}$$

$$\mathbf{H}^{\mathrm{T}} = \begin{bmatrix} \frac{\partial^2 f}{\partial^2 X_1} & \frac{\partial^2 f}{\partial X_1 \partial X_2} & \cdots & \frac{\partial^2 f}{\partial X_1 \partial X_n} \\ \frac{\partial^2 f}{\partial X_2 \partial X_1} & \frac{\partial^2 f}{\partial^2 X_2} & \cdots & \frac{\partial^2 f}{\partial X_2 \partial X_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 f}{\partial X_n \partial X_1} & \frac{\partial^2 f}{\partial X_n \partial X_2} & \cdots & \frac{\partial^2 f}{\partial^2 X_n} \end{bmatrix} \tag{9}$$

We consider $X_2 = X_2^C, X_3 = X_3^C, \ldots, X_n = X_n^C$ in Eq. (7), then $f(\mathbf{X})$ are simplified as: