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Adaptive isogeometric analysis in structural frames using a layer-based discretization to model spread of plasticity

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ABSTRACT

A distributed plasticity isogeometric frame model utilizing a layer-based discretization is formulated to capture the plasticity growth in large-deformation frames. In our formulation, B-spline basis functions are employed to define the deformation along the length, while a layer-based through-the-thickness discretization is adopted to capture the gradual plastification of the section. This separation of the thickness integration from the length direction enables the full 2D yielding development to be captured while maintaining a 1D data structure. The member-level geometrically nonlinear effects are also included. By introducing a continuity constraint in between two patches, rigid connection between two members is achieved in a multi-patch analysis setting. The formulation includes an adaptive analysis in which knots are inserted based on yield locations. In comparison to conventional layer-based finite elements, fewer degrees of freedom are needed to achieve the same level of accuracy due to the high-order smoothness of B-splines. Compared to existing isogeometric beam elements, the appealing feature is its capability of adaptively capturing the 2D spread of plasticity while maintaining a 1D data structure. The performance of the proposed model is assessed through several numerical examples involving gradual yielding of beams and frames under small and large deformations.

1. Introduction

Nonlinear analysis of structural frames is important in civil engineering for determining the load-displacement response of structures under extreme loads. A significant amount of research has been conducted over the past few decades to formulate beam/frame elements that can handle material and geometric nonlinearities. Material nonlinearities are typically handled by either lumped plasticity or distributed plasticity (i.e., layer) models [1]. The former is more computationally efficient, whereas the latter captures the spread of plasticity in a more accurate manner. Corotational formulations [2] are commonly employed to handle large displacements. While most of the work has focused on displacement-based formulations, recent studies have explored force-based and mixed formulations for improved accuracy [3–8]. While offering improved accuracy and a reduction in the number of degrees of freedom needed to model a structure, force-based elements require computationally expensive state determination algorithms to back-calculate stresses from nodal displacements. Thus, while software for the analysis of structural frames is well-developed and widely used in practice, existing displacement-based, force-based, and mixed element formulations have considerable limitations.

Isogeometric analysis (IGA) has gained significant attention in recent years as a novel computational method that integrates computer-aided design and analysis. It was first introduced by Hughes et al. [9] and has been applied to the analysis of solids, structures and fluids. IGA utilizes Non-Uniform Rational B-splines (NURBS) to represent the geometry as well as to describe the field variables. Thus, CAD drawings can be directly imported into finite element analyses without converting the geometry. While IGA was introduced to streamline the design process for complicated geometries, it has been shown to offer improvements in analysis for even simple geometries, as is shown in this paper. Readers are advised to refer to the original paper by Hughes et al. for a comprehensive overview of IGA. We provide a very brief review of the fundamental concepts for clarity.

In FEA, Lagrange basis functions are mapped into a single element's domain, and the finite elements are then assembled to arrive at the governing equilibrium equations. In IGA, however, the B-spline parameter space is defined over the entire patch, which is usually comprised of multiple elements. The parameter space is segmented into several elements by a non-decreasing set of coordinates called a knot vector. When the knots are equally

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spaced in the parameter space, the knot vector is considered uniform; otherwise it is non-uniform. A B-spline basis function is C^{p-1} continuous at a single knot, and C^{p-m} continuous at a repeated knot, where p is the degree of polynomial and m is the multiplicity of knots. The B-spline basis functions are computed based on the Cox-de Boor recursion formula [9]. With ξ being the natural coordinate, the basis function for $p = 0$ is

$$N_{i,0}(\xi) = \begin{cases} 1 & \xi_i \leqslant \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and for $p = 1, 2, 3, \ldots,$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \tag{2}$$

The B-splines that form the basis of IGA have the following properties:

- Partition of unity, i.e., $\sum_{i=1}^{n} N_{i,p}(\xi) = 1$
- Linear independence, i.e., $\sum_{i=1}^{n} a_i N_{i,p}(\xi) = 0 \Longleftrightarrow a_i = 0, i = 1, 2, \ldots, n$
- Non-negativity over the entire domain
- Local support, i.e., the basis function is non-zeros only in the domain $[\xi_i, \xi_{i+p+1}]$

A B-spline curve of polynomial degree p can then be constructed by the linear combination of control points P_i and its respective basis functions:

$$C(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) P_i \tag{3}$$

An advantage of IGA lies in the fact that all degrees of freedom (DOFs) are displacement-based, meaning that elements are rotation-free. This presents a significant opportunity for reducing the size of stiffness matrices for large structural systems, thereby offering significant savings in computational time. Moreover, recent studies [10–12] on IGA have indicated that the use of NURBS basis functions give improved accuracy over conventional finite element analysis (FEA) for certain applications.

Isogeometric analysis of beams has been studied by a number of researchers, and related papers cover shape optimization of beams [13], locking-free Timoshenko beams [14–16] and Kirchhoff-Love space rods [17]. Recent developments include the isogeometric analysis of plane-curved beams [18], which was based on the Timoshenko beam theory, and an isogeometric collocation method for thin beams and plates [19]. An implicit G^1 multi-patch Kirchhoff-Love space rod was contributed by Greco and Cuomo [20], in which the displacements of the first and last control points within one patch were decomposed using polar coordinates to obtain an automatic non-singular stiffness matrix. More recently, a shear deformable isogeometric beam using a single-variable formulation was developed [21].

While isogeometric analysis of beams has been shown to yield accurate results, the majority of previous models are in the range of elastic analysis and few have applied IGA to study the inelastic response of beam/frame structures loaded beyond yielding. The use of IGA for material nonlinear problems [22–26] has focused on the study of 2D and 3D continuums, which requires a tensor product of NURBS basis functions in multiple directions and therefore an excessive number of DOFs has to be introduced. Recently, a nonlocal damage theory was applied to study the inelastic behavior of beams [27]. However, this method involves in computing the six-order derivative of the nonlocal integral operator, which is computationally expensive. Additionally, it is not able to predict the full plasticity growth of the section.

Within this context, we propose a layer-based distributed plasticity isogeometric frame model based on the Euler-Bernoulli beam theory. Member-level geometrically nonlinear effects are accounted for through the inclusion of high-order strain terms. The formulation is therefore suitable for moderate deformations and rotations. The novelty and strength of this paper are:

- Instead of using tensor-product splines, the 1D B-spline basis function is used to represent the parametric domain in the length direction. The separation in integrating the layer-based thickness direction from the length direction significantly reduces the size of system matrices.
- Utilizing a layer-based discretization in the through-the-thickness direction not only allows the gradual localized plastification of the section to be captured accurately, but it also serves as a gradient-based a posteriori error estimator for adaptive analysis in the sense that localized yielding results in sharp curvatures which, in turn, are a perfect indication of the need for refinement.
- Yielding information is collected at the integration points in the layer to guide the adaptive refinement process. Lobatto quadrature rule is recommended in lieu of traditional Gaussian quadrature because Lobatto quadrature includes integration points at the ends of elements where plastic hinges are most likely to occur (e.g., connections between beams and columns, boundary condition locations, mid-span of members).
- A G^1 (i.e., geometrically continuous) continuity constraint is implemented in between patches, thereby adding rotational stiffness at beam-column connections. The constraint equation overlaps one control point on each side of the patch interface and only allows for yielding in the neighborhood of connections, rather than in the connections themselves. Therefore, the beam-column connection maintains rigidity throughout the analysis and plastic hinges are only allowed to form in beams and columns. This is rather important from a realistic structural point of view as yielding occurs at beam-column connections in traditional FE beam elements.
- The use of B-spline basis functions yields a rotation-free discretization, which represents great computational saving as compared to FEA.

2. Formulation

In this section, the governing equations of the distributed plasticity isogeometric frame model are presented. The model is "rotation-free" in the sense that the displaced shape is defined entirely in terms of the horizontal and vertical translations, u_i and v_i, respectively, at the n control points, as illustrated in Fig. 1. To enable material-nonlinear analysis, the patch is discretized into m layers [28], as shown in Fig. 2. It is assumed that the thickness of each layer is relatively small such that stress and strain are lumped at each layer across the section.

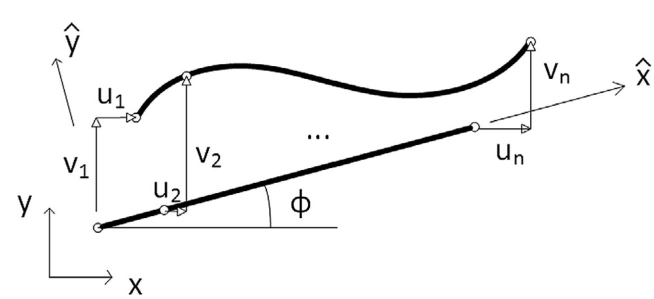

Fig. 1. Degrees of freedom of a layer-based IGA element.