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An efficient multi-time-step method for train-track-bridge interaction

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In this paper, the multi-time-step method (MTS) of time integration is proposed to reduce the computational cost of solving the dynamic interaction of a train-track-bridge coupled system (TTBS). Considering the different domain frequency characteristics of the dynamic responses of the train, track, and bridge, the MTS method decomposes the TTBS into two smaller subdomains: the train-track coupled subsystem with a high domain frequency, and the bridge subsystem with a low domain frequency. A fine time-step and a coarse time-step are respectively adopted for the train-track subsystem and the bridge subsystem to improve the computational efficiency. The two subsystems are coupled by the interaction forces between the track and bridge. Two partition types of the TTBS are introduced and the effect of different decomposition types on the accuracy and efficiency of the MTS method are discussed. The proposed method is validated by comparing the numerical results with field measurement data of a simply supported bridge. A numerical simulation of a train traversing a long-span cable-stayed bridge is used to demonstrate the computational efficiency and accuracy of the proposed method. It is shown that the proposed method is accurate and computationally more efficient than using a uniform time-step for the entire TTBS.

1. Introduction

With the increase in railway bridge spans and train speeds, the dynamic interaction between bridge and train has been regarded as an important factor that should be checked to ensure the serviceability of bridges and the running safety and ride comfort of trains [1–3]. Previous studies demonstrated that, for more accurate assessment of the running safety and passenger comfort of trains on bridges, the dynamic behavior of the track, as well as that of the train and the bridge, should be considered [4,5]. Meanwhile, compared with the analytical and semi-analytical methods for tracks [6] and bridges [7,8], the finite element (FE) method is a rigorous tool for detailed modeling of complex track and bridge structures [9]. Although the dynamic responses of all components in the train-track-bridge coupled system (TTBS) can be directly determined through the detailed FE model, the computational efficiency is considerably reduced when generating a better understanding of the dynamic performance of the entire system.

In previous studies of train-bridge interaction [10–12], the mode superposition method (MSM) has usually been applied to model the bridge, enhancing computational efficiency since it reduces considerably the number of degrees of freedom (DOFs). However, when a sophisticated track structure is involved in the train-bridge interaction model, strong local dynamic behaviors and high-frequency vibrations occur and it becomes difficult to determine how many modes of vibration should be involved in the analysis to simultaneously ensure accuracy and computational efficiency [10,13]. To consider the dynamic behavior of the tracks, therefore, the entire system is usually modeled using the direct stiffness method (DSM) [9,14–16]. In that case, when a bridge is rigorously modeled, the many DOFs generated are unavoidable, and the entire system becomes cumbersome. By combining the advantages of the DSM and the MSM, a hybrid track-bridge model for analysis of the train-track-bridge interaction was proposed by Yang et al. [17], with the tracks modeled by applying the DSM and the bridge modeled by applying the MSM.

To date, conventional numerical methods for solving the equations of motion of the TTBS using a uniform-time-step (UTS) for the entire structural problem domain. The size of UTS governed by the stability and accuracy requirements of the wheel-rail contact

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domain is confined to 10^{-4} s [6]. Obviously, using a UTS integration for the entire system is not appropriate because such a small time-step would lead to high computational cost, especially for the dynamic problem of train travelling through a long-span bridge which involves hundreds of thousands of degrees of freedom (DOFs) and time steps of structural responses. In fact, the dominant frequencies of different components of the TTBS show a striking contrast. For instance, the dominant frequencies of the train, bridge, and track subsystems are of the order of 1 Hz, 10 Hz, and 1000 Hz [18]. According to Clough and Penzien [19], when the time-step is less than or equal to 1/10 of the period corresponding to the upper bound frequency of the structure, the vibration responses can be captured accurately. Therefore, from the computational efficiency perspective, it is necessary to apply different time-step sizes to solve the equations of motion of different subsystems of the TTBS.

In fact, the multi-time-step (MTS) method, dividing the structure domain into several smaller subdomains with different time-steps, has proven remarkably successful in improving the efficiency of complex rigid-flexible problems with different dominant frequencies [20]. Since the pioneering works of Hughes and Liu [21] and Belytschko and Mullen [22], a number of studies published in the literature focused on the hybrid time integration scheme coupling (implicit/explicit) with homogeneous or heterogeneous time-steps [22–25]. The multi-time-step features of these approaches often suffer from stability difficulties when the subcycling is activated. This is due to, the need of interpolating values at the interface from the large time-step subdomain for computing the kinematic quantities in the fine time-step subdomain. Gravouil and Combesure [26] proposed a new MTS method, named the GC method, that incorporates the finite element tearing and interconnecting (FETI) technology developed by Farhat and Roux [27]. The GC method is based on a reduced interface problem derived from the velocity continuity at the interface for the finest time scale, and enables any Newmark time integration schemes could be coupled with different time steps in each subdomain. Prakash and Hjelmstad [28] refine the GC method by assuming velocity continuity at the large time-step instead of the fine time-step (PH method). Their method enables the dissipative drawback of the GC method to be tackled while optimizing the computation time related to solving the interface problem. The GC method and the PH method are both stable and accurate [26,28]. However, in these two methods, the kinematic quantities of the structure are treated as the sum of two kinds of quantities. One is calculated from the external forces only (the free problem) and the other is calculated from the interface reactions only (the link problem). Therefore, two sets of vibration equations and the interface reactions need to be solved and this leads to considerable computational effort. Especially, when the high-frequency domain of the structure contains a large number of DOFs, such as the train-track-bridge coupled system (TTBS), it often requires considerable computation time when the GC or PH method is used.

In this paper, a novel MTS method is presented to improve the computational efficiency of the dynamic interaction of coupled train-track-bridge systems. In this approach, the TTBS is divided into two subsystems – the train-track subsystem and the bridge subsystem. Fine time-steps are used to solve the equations of motion of the train-track subsystem because of its high-frequency vibration characteristics; while coarse time-steps are adopted for the bridge subsystem. In this study, the train on the bridge is composed of a sequence of identical vehicles and each 4-axle vehicle is modeled by a 10-degree-of-freedom dynamic system. The 3-D track and bridge model are established by the FE direct stiffness method. The equations of motion for the train-track subsystem are derived by assuming the linear Hertzian wheel-rail contact and using the track irregularity as the system excitation. These subsystems are coupled by enforcing the compatibility of the forces at the contact points between the track and the bridge. The proposed approach is validated to be appropriate by field measurement data obtained from a simply-supported girder bridge for heavy-haul trains in China. Finally, a long-span cable-stayed bridge under construction in China is selected for a numerical case study, in which the effects of different partitions of the TTBS and different time-steps on the efficiency and accuracy of the proposed MTS method are discussed. The results show that the proposed method can reduce computation time while still achieving high precision.

2. Train-track-bridge model

A fundamental model of the train-track-bridge dynamic interaction can be established based on the mechanism illustrated in Fig. 1. Although specific models for different high-speed trains or tracks and for different bridge structures can be very different [13,29], they all have the same basic framework that takes into account the train, track, and bridge subsystem components coupled with the wheel-rail interaction and the track-bridge interaction. For simplicity and clarity while introducing the MTS method and its application in train-track-bridge interaction dynamic analyses, the train is limited to in-plane motion and the wheel-rail contacts, which are modeled by Hertzian springs, are limited to the vertical, or Z direction (as shown in Fig. 1).

The vehicle elements of the train are discretized into the following main rigid bodies: one car body, two bogies, and four wheelsets, as shown in Fig. 1. The connections between the car body and each bogie and those between each bogie and its wheelsets are represented by linear springs and viscous dashpots. Under this assumption, stiffness, damping, and mass matrices with 10 DOFs can be constructed in the same manner as suggested by Biondi et al. [30].

The track and bridge can be accurately modeled using various FE types. The choice of each element type depends on the particular bridge configuration [31]. As shown in Fig. 1, the rail and sleepers are modeled as beam elements. The elasticity and damping properties of the fastener and ballast are modeled using uniaxial spring-dashpot units. The non-structural mass of the ballast bed is added to the self-weight of the bridge.

The wheel-rail interaction model is a key issue for the coupled train-bridge dynamic system. Several methods have been employed to address this issue, such as the corresponding assumption [32] and the Hertzian elastic contact theory [33] for analyzing the vertical wheel-rail interaction. Herein, the Hertzian stiffness k_h linearized from Hertzian non-linear contact theory [34] is used to couple the train and rail and can be written as:

$$k_h = \frac{3}{2G} P_0^{1/3} \tag{1}$$

where G represents the wheel-rail contact constant $(\mathrm{m/N^{2/3}})$ and P_0 represents the static wheel axle load. It should be noted that the vehicles are assumed to undergo in-plane motion while the track and bridge are established as 3D spatial models. Therefore, each wheelset is connected with two rails by two identical Hertzian springs, as shown in Fig. 1.

Because the train, track, and bridge are coupled as an integrated time-dependent system, the equations of motion of the train-track-bridge system can be written as: