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Towards improving the enhanced Craig-Bampton method

Seung-Hwan Boo, Jeong-Ho Kim, Phill-Seung Lee *

Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-gu, Daejeon 34141, Republic of Korea

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ABSTRACT

In this study, we improve the performance of the enhanced Craig-Bampton (ECB) method. The improved ECB method is derived by employing the algebraic substructuring and interface boundary reduction. Unlike for the original method, the residual substructural modes are compensated only for the reduced mass matrix, and this is the most attractive feature of the proposed method to reduce the computation time significantly. In addition, for effective implementation and computer memory management, we give a computer-aided formulation of the reduced mass, stiffness, and transformation matrices. Several large structural FE models are used to illustrate the significantly improved solution accuracy and computational efficiency of the improved method.

1. Introduction

Over the last half-century, component mode synthesis (CMS) [1–18] has frequently been employed in structural dynamics as an efficient and powerful tool to analyze the dynamic response of large finite element (FE) models with small computational effort. The primary concept of CMS is substructuring, and because of this, CMS is often called a dynamic substructuring method [19–21]. The first CMS method was proposed by Hurty in 1965 [1], and shortly thereafter, the Craig-Bampton (CB) method [2] was developed, and is the most popular CMS method at present. Later, various CMS methods were developed based on the CB method, and have been applied in many engineering fields [22–25].

In the CB method [2], using substructural eigenvalue problems, the substructural normal modes are computed, and those are classified into dominant and residual substructural modes using mode selection methods [22,26–28]. Then, the constraint modes are computed to define the static deformation between the substructures and the interface boundary [2]. Finally, a reduced model is constructed by synthesizing the dominant substructural modes, a very small portion of the total substructural modes, and the constraint modes.

In general, the error between the global (original) and reduced models is caused by the residual substructural modes that are neglected. Based on this fact, the residual substructural modes can be regarded as a crucial ingredient for improving the solution accuracy of the reduced model. Recently, using the residual flexibility matrix to compensate for the residual substructural modes, the enhanced Craig-Bampton (ECB) method [29,30] was developed. The ECB method results in a greatly improved reduced model in aspect of the solution accuracy.

However, the ECB method has limitations for solving large FE models involving more than hundreds of thousands of degrees of freedom (DOFs). In the ECB method, the substructuring is accomplished by using the physical domain-based substructuring considering the geometrical characteristics of the structure. Therefore, it is not easy to make a large number of substructures. In such cases, each substructure may contain relatively large DOFs and thus the computation time for calculating the constraint modes, which requires the computation of the inverse of a matrix, would be considerably expensive. In addition, because the residual flexibility matrix, a key to the ECB method, is highly populated, thus it has substantial memory requirements and requires restricting computational work during the reduction procedure. For these reasons, the original ECB method is not appropriate for dealing with large FE models. Given the recent trend of increase in the size of FE models, these limitations should be resolved.

In this study, to resolve the aforementioned limitations, we improve the performance of the ECB method. We first identify sources deteriorating the computational efficiency in the original method, and focus on managing them effectively in the proposed method. To increase the computational efficiency and reduce the requirement for computer memory, we use algebraic substructuring [31–38], giving many small substructures, instead of physical domain-based substructuring. To reduce the size of the interface boundary, inevitably increased by algebraic substructuring, interface boundary reduction [39] is employed. In the improved ECB

* Corresponding author.
E-mail address: phillseung@kaist.edu (P.-S. Lee).

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method, the residual flexibility matrix, reflecting the residual substructural modes, is applied only to the reduced mass matrix. This is a significant feature for reducing the computation time. We also demonstrate a computer-aided formulation for efficient implementation and computer memory management of the proposed method.

In Section 2 of this paper, the original ECB method is reviewed in brief. In Section 3, the improved ECB method is derived, and the computer-aided formulation is presented in Section 4. In Section 5, we verify the performance of the proposed method through several large structural FE models, and finally, conclusions are drawn in Section 6.

2. Original ECB method

In this section, we briefly review the formulation of the original ECB method. The detailed derivation procedure is described in Refs. [29,30].

The generalized eigenvalue problem for the non-partitioned global (original) structural FE model is defined by

$$\mathbf{K}_g\mathbf{u}_g = \lambda\mathbf{M}_g\mathbf{u}_g, \tag{1}$$

where $\mathbf{M}_g$ and $\mathbf{K}_g$ denote the mass and stiffness matrices for the global structure non-partitioned, respectively, and $\mathbf{u}_g$ and λ denote the global displacement vector and the eigenvalue of the global structure, respectively.

Through the substructuring shown in Fig. 1, the global structure is partitioned into n substructures that are fixed to its interface boundary. Then, Eq. (1) can be represented in a partitioned matrix form as

$$\begin{bmatrix} \mathbf{K}_s & \mathbf{K}_c \\ \mathbf{K}_c^T & \mathbf{K}_b \end{bmatrix}\begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \lambda\begin{bmatrix} \mathbf{M}_s & \mathbf{M}_c \\ \mathbf{M}_c^T & \mathbf{M}_b \end{bmatrix}\begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix}, \tag{2}$$

in which the subscripts s, b, and c denote the substructural, interface boundary, and coupled quantities, respectively. $\mathbf{M}_s$ and $\mathbf{K}_s$ are block-diagonal mass and stiffness matrices of which diagonal terms consist of substructural mass and stiffness matrices, $\mathbf{M}_s^{(i)}$ and $\mathbf{K}_s^{(i)}$ (for $i = 1, 2, \ldots, n$), respectively.

In the CB method, the transformation matrix is defined by

$$\mathbf{T}_0 = \begin{bmatrix} \boldsymbol{\Phi}_s & \boldsymbol{\Psi}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix} \quad \text{with} \quad \boldsymbol{\Phi}_s = \begin{bmatrix} \boldsymbol{\Phi}_s^d & \boldsymbol{\Phi}_s^r \end{bmatrix}, \quad \boldsymbol{\Psi}_c = -\mathbf{K}_s^{-1}\mathbf{K}_c, \tag{3}$$

where $\boldsymbol{\Phi}_s$ denotes the substructural eigenvector matrix containing all substructural modes, and it is decomposed into $\boldsymbol{\Phi}_s^d$ and $\boldsymbol{\Phi}_s^r$, which are corresponding to the dominant and residual substructural modes, respectively. Here, $\boldsymbol{\Psi}_c$ and $\mathbf{I}_b$ denote the constraint mode matrix and the identity matrix for the interface boundary, respectively.

In Eq. (3), $\boldsymbol{\Phi}_s$ is a block-diagonal matrix, of which the diagonal terms consist of the substructural eigenvector matrices $\boldsymbol{\Phi}_s^{(i)}$ (for $i = 1, 2, \ldots, n$), and $\boldsymbol{\Phi}_s^{(i)}$ are computed from the following substructural eigenvalue problems

$$\begin{aligned} &\mathbf{K}_s^{(i)}\boldsymbol{\Phi}_s^{(i)} = \mathbf{M}_s^{(i)}\boldsymbol{\Phi}_s^{(i)}\boldsymbol{\Lambda}_s^{(i)} \quad \text{with} \quad \boldsymbol{\Phi}_s^{(i)} = [\boldsymbol{\Phi}_d^{(i)}\boldsymbol{\Phi}_r^{(i)}], \\ &\boldsymbol{\Lambda}_s^{(i)} = \begin{bmatrix} \boldsymbol{\Lambda}_d^{(i)} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_r^{(i)} \end{bmatrix} \quad \text{for} \quad i = 1, 2, \ldots, n, \end{aligned} \tag{4}$$

in which $\boldsymbol{\Lambda}_s^{(i)}$ denotes the substructural eigenvalue matrix corresponding to the i^{th} substructure, and $\boldsymbol{\Phi}_s^{(i)}$ and $\boldsymbol{\Lambda}_s^{(i)}$ are decomposed into dominant terms ($\boldsymbol{\Phi}_d^{(i)}$ and $\boldsymbol{\Lambda}_d^{(i)}$) and residual terms ($\boldsymbol{\Phi}_r^{(i)}$ and $\boldsymbol{\Lambda}_r^{(i)}$).

The constraint mode matrix $\boldsymbol{\Psi}_c$ in Eq. (3) is computed by

$$\boldsymbol{\Psi}_c = \begin{bmatrix} \boldsymbol{\Psi}_c^{(1)} \\ \vdots \\ \boldsymbol{\Psi}_c^{(n)} \end{bmatrix} \quad \text{with} \quad \boldsymbol{\Psi}_c^{(i)} = -\left(\mathbf{K}_s^{(i)}\right)^{-1}\mathbf{K}_c^{(i)} \quad \text{for} \quad i = 1, 2, \ldots, n, \tag{5}$$

where $\boldsymbol{\Psi}_c^{(i)}$ denotes the i^{th} substructural constraint mode matrix. Here, the inverse matrix $\left(\mathbf{K}_s^{(i)}\right)^{-1}$ can be effectively computed using the Cholesky factorization of $\mathbf{K}_s^{(i)}$.

The global displacement vector $\mathbf{u}_g$ is transformed using the transformation matrix $\mathbf{T}_0$ in Eq. (3) as follows

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \mathbf{T}_0\mathbf{u} \quad \text{with} \quad \mathbf{T}_0 = \begin{bmatrix} \boldsymbol{\Phi}_s^d & \boldsymbol{\Phi}_s^r & \boldsymbol{\Psi}_c \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_b \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \mathbf{q}_s^d \\ \mathbf{q}_s^r \\ \mathbf{u}_b \end{bmatrix}, \tag{6}$$

in which $\mathbf{u}$ denotes the generalized coordinate vector, and $\mathbf{q}_s^d$ and $\mathbf{q}_s^r$ denote the modal coordinate vectors corresponding to the dominant and residual substructural eigenvector matrices, $\boldsymbol{\Phi}_s^d$ and $\boldsymbol{\Phi}_s^r$, respectively.

In Eq. (6), selecting the dominant terms, $\boldsymbol{\Phi}_s^d$ and $\mathbf{q}_s^d$, we can obtain the approximated global displacement vector $\overline{\mathbf{u}}_g$ as

$$\mathbf{u}_g \approx \overline{\mathbf{u}}_g = \overline{\mathbf{T}}_0\overline{\mathbf{u}} \quad \text{with} \quad \overline{\mathbf{T}}_0 = \begin{bmatrix} \boldsymbol{\Phi}_s^d & \boldsymbol{\Psi}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}, \quad \overline{\mathbf{u}} = \begin{bmatrix} \mathbf{q}_s^d \\ \mathbf{u}_b \end{bmatrix}, \tag{7}$$

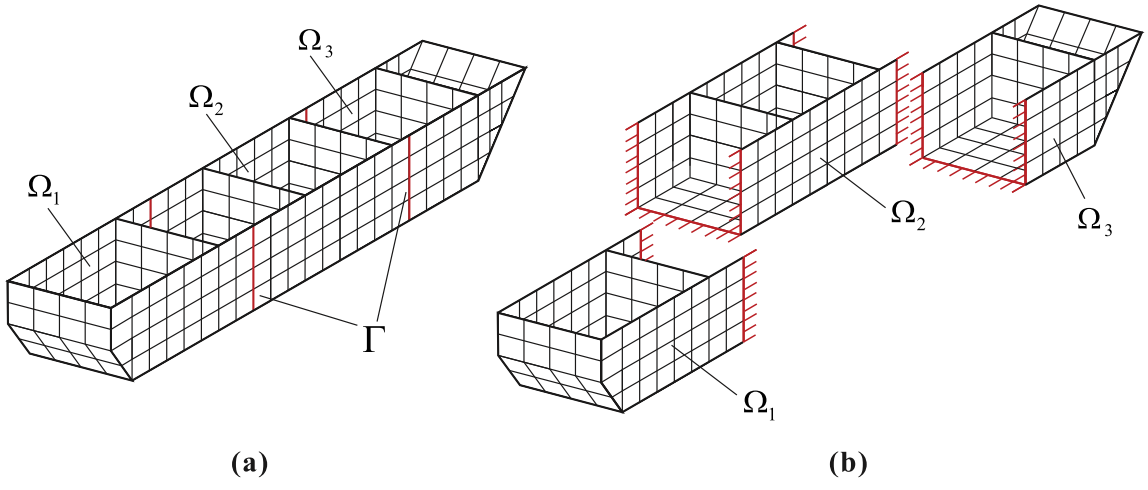

Fig. 1. Substructuring for the global structure: (a) Partitioned structure, where Ω_i and Γ denote the i^{th} substructure and the interface boundary, respectively, (b) fixed interface boundary.