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Complete monotonic expression of the fourth-moment normal transformation for structural reliability

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ABSTRACT

Probability distributions of basic random variables are essential for the accurate evaluation of structural reliability. In engineering practice, the probability distributions of some random variables are often unknown and the only available information about these may be their statistical moments. To conduct structural reliability analysis without the exclusion of random variables with unknown probability distributions, the fourth-moment normal transformation (FMNT) has been proposed. However, the applicability of expression of the FMNT has not been sufficiently investigated. Furthermore, the monotonic regions of the FMNT are not defined without which the application of the transformation is inconvenient, or even unreliable in reliability analysis. In the present paper, a complete expression of the FMNT including six cases with different combinations of skewness and kurtosis is derived, and the monotonicity of each case of the FMNT expression is confirmed. Literature suggests that the complete monotonic expression of the fourth-moment normal transformation is the first time to be successfully accomplished up to date. Through the numerical examples, the FMNT is found to be quite efficient for normal transformation and to be sufficiently accurate to include random variables with unknown probability distributions in structural reliability analysis.

1. Introduction

Searching efficient approaches for the probability of failure of structures has led to the development of various approximation methods. For almost all current reliability methods, such as FORM [1,2], SORM [3–5], the importance sampling Monte Carlo simulation (MCS) [6,7], the method of moments [8,9], the basic random variables are assumed to have known probability density functions (PDFs) or cumulative distribution functions (CDFs). With the known CDFs/PDFs, the normal transformation (the *x-u* transformation) and its inverse transformation (the *u-x* transformation) can be realized using the Rosenblatt [10] or Nataf transformations [11]. However, in many practical engineering problems, the distributions of some basic random variables are often unknown due to the lack of statistical data. In such circumstances, the Rosenblatt transformation or Nataf transformation cannot be applied, and a strict evaluation of the probability of failure is not possible. Thus, an alternative measure of reliability is required.

A comprehensive framework for the analysis of structural reliability under incomplete probability information was proposed by Der Kiureghian and Liu [12] based on the Bayesian idea. Zong and Lam [13] suggested a method of estimating complex distributions using B-spline functions, in which the estimation of the PDF is summarized as a nonlinear programming problem. Using the statistical data, the probability distributions of random variables can also be estimated by non-parametric approach such as the kernel density estimation (KDE) [14–17]. Because the first four moments (mean, standard deviation, skewness, and kurtosis) having clear physical definitions are common in engineering and can be easily obtained using the sample data, the *u-x* and *x-u* transformations realized using the first four moments of the random variables will be focused on in this paper.

One method to realize the transformation based on the first four moments is using the distribution families. The distribution families, such as the Pearson system and the Burr system [18], can be used to estimate the distributions of the random variables. Although these systems are very flexible, they are difficult to implement, in particular, at the artificial interfaces between different distribution types. Low [19] proposed a shifted generalized lognormal distribution (SGLD). Although this distribution possesses

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Notation

A, B	coefficient of the expression of the FMNT
A_s, A_c	cross-sectional area of steel and concrete, respectively
a, c	coefficients of p, q, J_1^*, J_2^*
a_h, b_h, c_h	coefficients of the Winterstein formula
a_1, a_2, a_3, a_4	polynomial coefficients used in the third-order polynomial expression
a_2', a_3'	polynomial coefficients of Eq. (2a)
c_p, c_s	damping coefficients of the system in Example 5 for the primary and secondary oscillators, respectively
D_{WT}	deadweight tonnage of the vessel
E	modulus of elasticity
E_s, E_c	modulus of elasticity of steel and concrete, respectively
$F(\cdot)$	cumulative distribution function of a random variable
$f(\cdot)$	probability density function of a random variable
$G(\cdot)$	performance function
h_3, h_4	coefficients of the Winterstein formula
J_0	stationary point of a cubic function when the cubic function is monotonic
J_1 and J_2	stationary points of a cubic function when there are three separate monotonic regions
J_1^* and J_2^*	coefficients for determining the number of real solutions to Eq. (2a)
l	length of the bars
m_p, m_s	mass of the system in Example 5
k_p, k_s	spring stiffness of the system in Example 5 for the primary and secondary oscillators, respectively
K	coefficient of Eq. (11)
K_L, K_R	coefficients of spring
P_f	failure probability
p, q, r	coefficients of the expression of u
p_s	peak factor of the system in Example 5
q_r	uniformly distributed load
S_0	intensity of the white noise
$S_u(\cdot)$	third polynomial of the standard random variable
$S_u^*(\cdot)$	simplified third polynomial of the standard random variable
u	standard normal random variable
u_a	allowable displacement in Example 2
u_{max}, u_{med}, u_{min}	maximum, medium, and minimum values of u, respectively
V	design impact velocity
V_T	typical impact velocity
$V_{\min}$	minimum design impact velocity
$\mathbf{X}$	array of random variables
x	random variable
x_0	distance to face of pier from centerline of channel
x_c	distance from centerline of channel to edge of channel
x_L	three times the overall length of the vessel
x_{limit}	the limitation of x in Example 4
x_i'	possible value of x'
x_s	standardized random variable
x'	transformed random variable
x_1', x_2', x_3'	values of x'
α_i, β_i (i = 1, 2, 3, 4)	parameters of the distribution in Example 1
α_{3x}	skewness of a random variable
α_{4x}	kurtosis of a random variable
β_{FORM}	first-order reliability index
β_{SORM}	second-order reliability index
β_{MCS}	reliability index obtained by Monte Carlo simulation
Δ	coefficient of the expression of the FMNT
Ω	water level
Θ	impact angle
$\Phi(\cdot)$	cumulative distribution function of a standard normal random variable
$\phi(\cdot)$	probability density function of a standard normal random variable
γ	mass ratio of the system in Example 5
μ_x	mean value
μ_{mp}	mean value of the mass of the system in Example 5
σ_x	standard deviation
θ	coefficient of the expression of u
θ_{sr}	a tuning parameter in Example 5
τ	duration of loading
ω_a	average frequency of the system in Example 5
ω_p, ω_s	natural frequencies of the system in Example 5 for the primary and secondary oscillators, respectively
ζ_a	damping ratio of the system in Example 5
ζ_p, ζ_s	damping ratios of the system in Example 5 for the primary and secondary oscillators, respectively
$\xi(\cdot)$	a function of x used in the Winterstein formula

many desirable advantages, it requires complicated computation [19].

Another method is to use polynomial transformations, in which the random variable x is directly expressed as a polynomial of a standard normal random variable u. Various such transformations have been proposed [20–23]. All the transformations (u-x transformation) are expressed as a third-order polynomial of u, which is generally formulated as follows [21]:

$$\frac{(x-\mu_x)}{\sigma_x} = x_s = S_u(u) = a_4u^3 + a_3u^2 + a_2u + a_1 \tag{1}$$

where x_s is the standardized random variable; μ_x and σ_x are the mean value and standard deviation of x, respectively; $S_u(u)$ is a third-order polynomial of u; and a_1, a_2, a_3, and a_4 are polynomial coefficients that can be obtained by making the first four moments of $S_u(u)$ equal to those of x_s [21], which are shown in details in Appendix A.

Since Eq. (1) is simple, the third-order polynomial transformation has been widely applied in structural reliability [23–31]. When the transformation is applied to structural reliability analysis, u-x transformation is relatively easy to conduct, which can be uniquely determined by the value of u. The inverse transformation (x-u transformation) should be conducted by finding the solution of Eq. (1), which defines a fourth-moment normal transformation (FMNT).

A single expression of the FMNT based on the Hermite moment model has been proposed by Winterstein [22], and it is widely used to transform extreme fractiles from non-Gaussian processes to Gaussian ones. Noting that the Hermite moment model has been constrained by a monotone limitation, Choi and Sweetman [28] offered an alternate solution technique to overcome the monotone limitations of the original Hermite moment model, and it was applied to highly skewed cases with near-Gaussian kurtosis, exampled by a tension leg platform (TLP) subject to irregular seas. However, for the third-order polynomial of u (Eq. (1)), such investigation hasn't been conducted. With different combinations of skewness and kurtosis, which result in different combinations of the parameters of Eq. (1), there may be more than one possible values of u corresponding to one x. Without clear definition of the complete expression of the FMNT and the corresponding monotonic regions of x or u, the FMNT will be inappropriate, even unreliable, to be used in structural reliability.

Therefore, the objectives of this paper are to derive the complete expressions of the FMNT with different combinations of